Statistical Properties of Diversity Factors for Probabilistic Loading of Distribution Transformers

V. P. Chatlani, *Student Member, IEEE*, D. J. Tylavsky, *Senior Member, IEEE*, D. C. Montgomery, and M. Dyer

***Abstract*--The diversity factor is an essential tool used for loading of distribution transformers. Typically, diversity factors are approximate values and treated as deterministic. By calculating many sample diversity factors it is possible to develop statistics that describe the diversity factor and use these statistic to perform probabilistic transformer loading. Results from work reported here confirm the independence of the diversity factors calculated by bootstrap sampling of measured residential load data. This paper shows that the diversity factor is not normally distributed as assumed in other work. The Anderson-Darling test shows that the diversity factor obeys a gamma distribution. The statistical nature of the predicted demand with the statistical nature of the transformer loading capability can be merged to arrive at a quantifiable statistical assessment of transformer loading.**

***Index Terms*—Diversity factors, residential load data**

I. Introduction

DIVERSITY factors are used by planning engineers to predict peak demand observed at a distribution transformer and select the transformers' MVA rating, or alternately, select the number of (residential) loads that may be connected to the transformer. If the incorrectly sized transformer is installed—or the number of loads connected too large—the demand peak at the transformer may be greater than the transformer's rating. Consistent overloading of the transformer will consume insulation life and reduce transformer life. Because perfect knowledge of neither the transformer rating nor the diversity factor exists, statistical properties of the transformer and diversity factor need to be used when sizing the transformer. The ultimate goal of the work reported here is to merge the statistical nature of the predicted demand with the statistical nature of the transformer loading capability. By merging these statistics, we arrive at a quantifiable statistical assessment of transformer loading and a measure of the risk associated with that loading. We will show how the probability density function, PDF, for the peak load has been determined and how it can be used with the PDF of the transformer capability to quantify risk.

Calculating diversity factors is not new. Diversity factor research has been completed for residential areas in Arkansas in [1], [2] and [11]. The work completed in [1] used data for 299 customers, which were divided into two groups based on the type of heating used by the customer. For Arkansas it was determined that customers with electric heating had different load profiles when compared to customers with non-electric heating.

Reference [11] presents the statistical properties of the diversity factors calculated in [1] and [2]. The diversity factor statistics were calculated for various group sizes (with 60 replicates for each group size), and the DF's were determined to be random and normally distributed.

In this paper we show that the normal distribution is a poor approximation to the DF's. Instead, we show that DF's more closely fit a gamma distribution. We show how the statistical model of the diversity factor along with a statistical model of a transformer can be used for statistical loading calculations.

II. Calculating The Diversity Factor

Diversity factors (DF's) are complicated metrics that depend on many variables. For example, DF's depend on customer classes: residential, commercial, light industrial and heavy industrial. DF's vary by month or season, and vary for different regions of the country.

In this analysis, we focus on DF's for residential loads of the arid southwest during the peak loading conditions, i.e., the summer season; however the techniques we use here can be extended to any region, any type of load and any season.

The diversity factor, defines the relationship between the peak of the group load and the sum of individual load peaks over a specified period of time [8], [9].

$$DF = \frac{\sum Individual\ Peaks}{Group\ Peak} \qquad (1)$$

where DF is the diversity factor which will vary based on the size and composition of the group.

Our goal is to calculate the probability distribution of the DF for various group sizes. We then wish to use these

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V. P. Chatlani and D. J. Tylavsky are with the Department of Electrical Engineering, Arizona State University, Tempe, AZ 85287 USA (e-mail: tylavsky@asu.edu).

D. C. Montgomery is with the Department of Industrial Engineering, Arizona State University, Tempe, AZ 85287 USA (e-mail: doug.montgomery@ asu.edu).

M. Dyer is with Salt River Project, Tempe, AZ 85281 USA (e-mail: michael.dyer@srpnet.com)

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statistics to perform probabilistic transformer loading.

To accomplish this goal, we obtained load data from Salt River Project (SRP) in Phoenix, Arizona. The raw data contained load data for 685 different residences for July, when the loads are the heaviest. The data consisted of customer load (MVA) averaged over fifteen-minute intervals. We did not have information about whether the homes in our sample had air conditioning units or swamp coolers. This would make some difference in the DF's calculated; however the difference is not expected to be great. Further, our study is aimed at sizing newly installed transformers and few if any new homes have swamp coolers installed.

To calculate the probability distribution of the DF's we took repeated samples of residence data with replacement (a.k.a. bootstrapping [12]). Two pieces of information were taken from these samples. We first found the individual peaks for the residences selected and added these to get the sum of the individual peaks. We next added the load profiles of all of the residences sampled and identified the group peak. We then applied (1) to calculate the diversity factor. This process was repeated n times, where n is the number of replicates in a sample of DF's. In this study, a large sample with 1000 replicates was used to calculate the DF for any desired group size, so that the DF's we calculated would be precise. From these samples we calculated the mean and the standard deviation. The means of the DF's are shown in Fig. 1.

This figure shows that minimum DF is 1 (as can easily be verified from (1)) and that the DF initially increases quickly as more customers are added to the group being observed. For larger group sizes, the diversity factor is less sensitive to number in the group. This behavior is consistent with that found in [8].

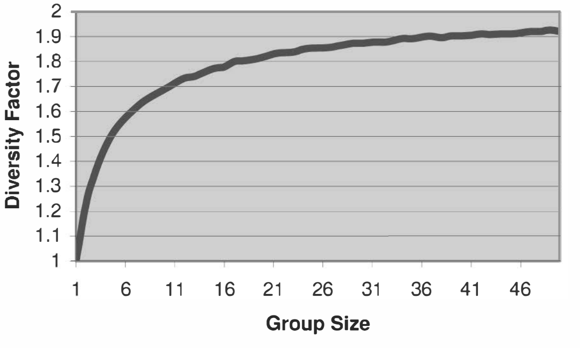

Fig. 1. Summer diversity factors for group sizes ranging from 1 to 50 for Phoenix, Arizona.

III. Sample Independence

The objective of our work is to determine the statistics of the DF so that they can be used in probabilistic loading of transformers. To know whether we can assign any type of distribution to the DF's we calculate in Fig. 1, we must first show that the observed values of the replicates of the DF's for each group size are independent and random; this will allow us to conclude that the replicates within the sample are not correlated.

The runs test is used to test if the observations within the samples are random independent variables [10]. The runs test can only use categorical data with two distinct cases. In the case of numerical data, any data points above the median are assigned to one category, and any data points below the median are assigned to a second category. Data points exactly equal to the median are ignored. In the runs test, a run is defined as a subsequence of like data points. If there are too few or too many runs within a sequence the hypothesis of randomness is rejected.

Since only 60 replicates for each group size were used in [11], in this work we use only 60 replicates of DF's for each group size so that our results were comparable.

Compared in TABLE I are the results of the runs test for the Arizona data and from the Arkansas data [11] for different group sizes. A zero in this table means the test for independence is passed; a one means the test for independence has failed.

TABLE I
Comparison of Runs Test Results for the Diversity Factor for Select Group Sizes for the Month of July with 60 Replicates

Group Size	Results from [11]	Results from this Work
2	1	0
5	1	0
10	0	0
15	0	0
20	0	0
25	0	0
30	1	0

The results presented in TABLE I, indicate that the observations in the sample are random and independent variables.

IV. Chi-Square Goodness-of-Fit Test

A. Testing Small Sample Sizes

After concluding that the observations are independent random variables, the Chi-Square Goodness-of-Fit Test was used to determine whether the samples were normally distributed. If a sample is normally distributed, the mean and variance of the sample can be used to determine a confidence interval, given a required significance level. Once again, the results for Arkansas are reproduced in TABLE II for easy comparison with the results produced for the Phoenix area. The results in TABLE II follow the same formatting as in TABLE I. The table consists of zeros and ones to indicate whether the Chi-Square Goodness-of-Fit test passes (the samples are normally distributed) or fails, where a numeric value of 1 indicates failure. The results produced by the two different studies are comparable.

B. Testing Large Sample Sizes

The Runs test and Chi-Square Goodness-of-Fit test conducted in the previous sections were repeated for a larger number of replicates to test whether the previous conclusions (about independence and normality) would hold true. The runs test for independence supported the same conclusion regardless of the number of replicates.

TABLE II
COMPARISON OF CHI-SQUARE GOODNESS-OF-FIT RESULTS FOR THE DIVERSITY FACTOR FOR SELECT GROUP SIZES FOR THE MONTH OF JULY WITH 60 REPLICATES

Group Size	Results from [11]	Results from this Work
2	1	0
5	1	1
10	1	0
15	0	0
20	0	1
25	0	0
30	0	0

Chi-Square Goodness-of-Fit test results for larger replicate sizes yielded results that contradicted the conclusions of normality. Fig. 2 and Fig. 3 show the results of the Chi-Square Goodness-of-Fit test for 100 and 1000 replicates respectively. Every bar in the graph indicates a point where the test for normality fails.

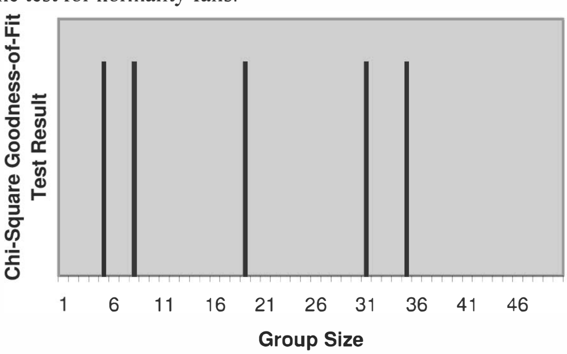

Fig. 2. Chi-Square Goodness-of-Fit test for different group sizes ranging from 1 to 50 where the diversity factor for each group size has 100 replicates.

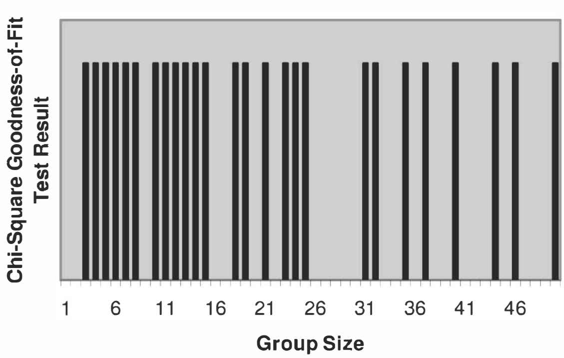

Fig. 3. Chi-Square Goodness-of-Fit test for different group sizes ranging from 1 to 50 where the diversity factor for each group size has 1000 replicates.

The test for normality produced acceptable results when the sample had 100 replicates. Out of 50 trials, the test failed a total of 5 times. For the larger sample size with 1000 replicates, the Chi-Square Goodness-of-Fit test failed 26 times out of 50. This indicates that the assumption of normality is suspect.

It was concerning that the DF distribution appears normal for a small number of replicates and not normal when the number of replicates is larger. It is well known that the Chi-Square Goodness-of-Fit test is the least powerful test because it depends on separating the data into arbitrary bins. We therefore turned to a test that is well known to be more powerful: the Anderson-Darling test [7]. A test is considered to be powerful if the probability of accepting the null hypothesis when the null hypothesis is wrong is low.

V. ANDERSON-DARLING TEST

The Anderson-Darling test is well known to be a more reliable indicator of normality than the Chi-Square Goodness-of-Fit test because it can be performed on both binned and unbinned data, and provides equal sensitivity at the tails as the median [7].

The Anderson-Darling test calculates a test statistic, which is a large value when the data does not fit the distribution to which it is being compared. A corresponding p-value is determined which is the probability of observing the given statistic or one more extreme, assuming that the null hypothesis is true. It is generally accepted that p-values greater than or equal to 0.05 indicate that there is not sufficient evidence to reject the null hypothesis. The null hypothesis in our case is: the distribution is normal. We applied the Anderson-Darling test to the DF data set generated by bootstrapping.

A. *Testing the Adequacy of the Normal Fit*

The Anderson-Darling test for normality was repeated for the samples with 100 replicates, and again for the samples with 1000 replicates. The results produced from this test are presented in Fig. 4 and Fig. 5. Each bar in the graph indicates a point where the test for normality fails.

In the case of 100 replicates, the test for normality failed 13 times out of 50 trials. Comparing Fig. 4 and Fig. 2, one can see that when using the Anderson-Darling test, the test for normality fails more often than when using the Chi-Square Goodness-of-Fit test.

Using the Anderson-Darling test, the test for normality failed 37 times when using 1000 replicates as shown in Fig. 5. From these results it can be concluded that the normal distribution is not the appropriate fit for the data.

A visual test for normality is the normal probability plot. For normally distributed data, the DF data points generated by the bootstrapping procedure should lie along the straight line of Fig. 6; instead, the tails of the DF distribution diverge—for 1000 replicates and a group size of 8. It is suspected that this divergence occurs because the DF has a lower bound of 1. (The DF is the ratio of the sum of the individual peaks divided by the group peak and will take on its minimum value of 1 when the group size is 1 or when all of the peaks of the loads in the group have coincident peaks.) When the number of replicates is small, the probability of have DF data points near the lower bound is small. As the number of replicates increases, we are more likely to encounter a combination of loads that have coincident peaks (or nearly coincident peaks) and see an accumulation of data points near the lower bound; this will cause the tails of the distribution to diverge as shown in Fig. 6 and cause the test for normality to fail.

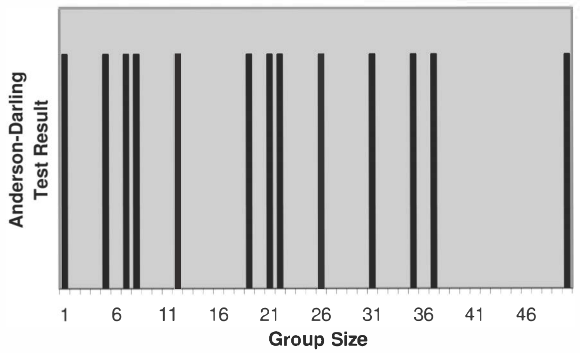

Fig. 4. Anderson-Darling test for different group sizes ranging from 1 to 50 where the diversity factor for each group size has 100 replicates.

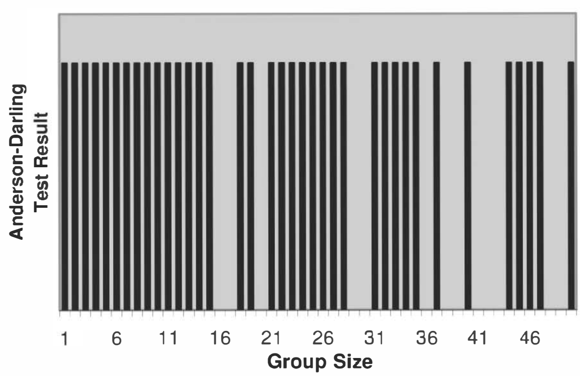

Fig. 5. Anderson-Darling test for different group sizes ranging from 1 to 50 where the diversity factor for each group size has 1000 replicates.

The weight of numerical evidence suggests that the DF's are not distributed normally. In addition, there is theoretical evidence to back this up. Recall that the normal distribution is not bounded; it ranges from negative infinity to positive infinity, while the distribution of DF's has a lower bound. Since the normal distribution is not bounded, it is unlikely that the normal distribution will fit DF data. Therefore, we chose to test whether a distribution with a lower bound might fit our data better.

B. Testing the Adequacy of the Gamma Fit

The gamma distribution may be a better fit for the diversity factors calculated because it has a lower bound of 0, and no upper bound. The DF's have a lower bound of 1 and no upper bound. To conform our data to the gamma distribution, it is necessary to subtract 1 from every DF data point generated by the bootstrapping procedure; we will call this shifting the DF data values.

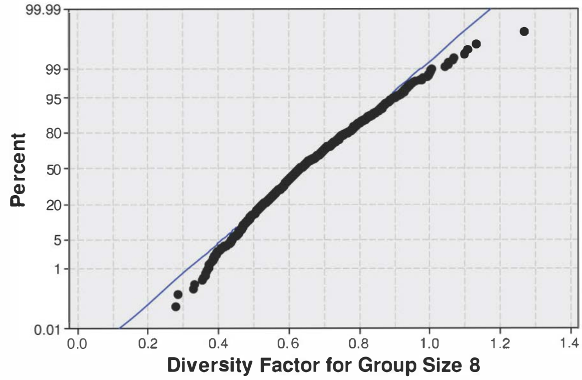

Fig. 6. Normal probability plot for the diversity factor for group size 8, with 1000 replicates.

From viewing the histogram of the data for group size 8 with 1000 replicates in Fig. 7, the difference in the normal and gamma fit may not seem significant, but comparing the p-values produces a more concrete and confident result. The p-value corresponding to the normal fit for this sample is less than 0.005, whereas the p-value corresponding to the gamma fit is greater than 0.25.

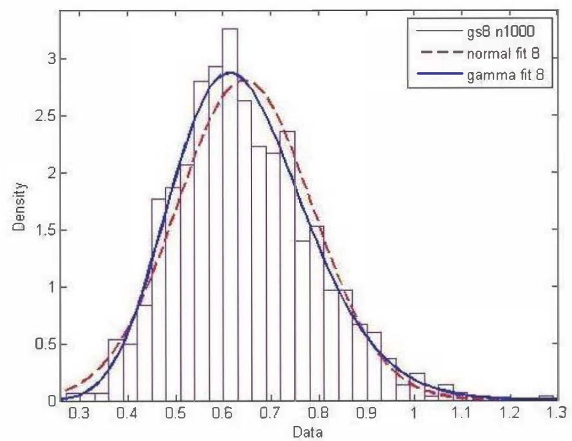

Fig. 7. Probability density function for 1000 replicates of the shifted diversity factors for group size 8, with the corresponding normal and gamma fit.

The shifted DF's for the different group sizes were fit to a gamma distribution using the Anderson-Darling test to determine if the fit was appropriate. The results for the Anderson-Darling test for the gamma distribution are presented in Fig. 8.

The smaller group sizes do not fit the gamma distribution very well. The gamma distribution begins to fit the diversity factors better for a group size of 7 and larger. The probability plot assuming a gamma distribution for a group size of 8 is presented in Fig. 9.

The probability plot for the gamma distribution (for a group size of 8 with 1000 replicates) shown in Fig. 9 fits the data better than the probability plot for the normal distribution. Similarly the DF's for the larger group sizes fit the gamma distribution better than the normal distribution. We concluded that the gamma distribution is a more appropriate model to use when characterizing the DF.

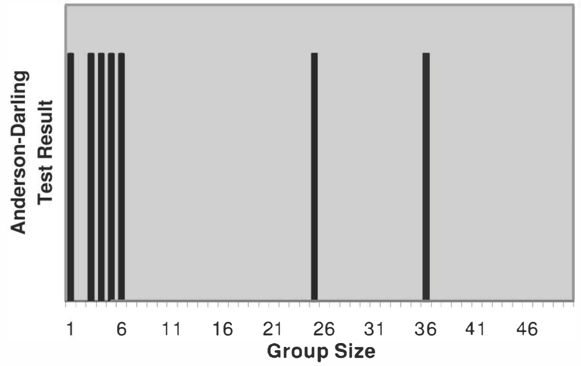

Fig. 8 Anderson-Darling test for the gamma distribution for different group sizes ranging from 1 to 50 where the diversity factor for each group size has 1000 replicates.

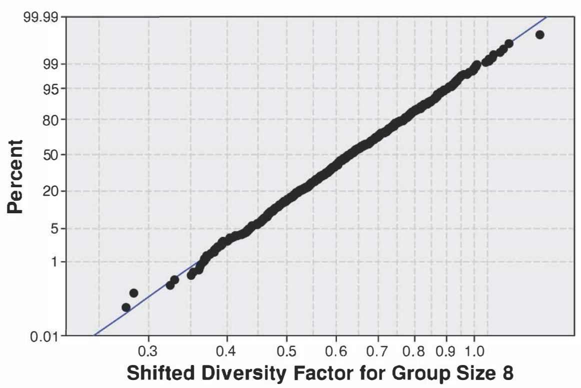

Fig. 9. Gamma distribution probability plot for the shifted diversity factor for group size 8, with 1000 replicates.

VI. Loading of Distribution Transformers

Presently, distribution transformer loading is based on limited and possibly out of date data. Incorrect transformer sizing is very costly to utilities both oversized and undersized. Quantifying the uncertainties in both transformer peak demand and peak loading capability, will allow utilities to size transformers more accurately with quantified risk assessment.

Since the DF depends on the data sample taken, the true diversity factor is not known, only its best estimate and its statistics. Likewise, the loading capability of distribution transformers, is not known exactly, but can be described statistically from measured data. For substation distribution transformers, [3]-[6], [13], the statistics of the ratings correspond to a normal density function. We assume in this work, as shown in Fig. 10, that the ratings of distribution transformers also correspond to normal density functions.

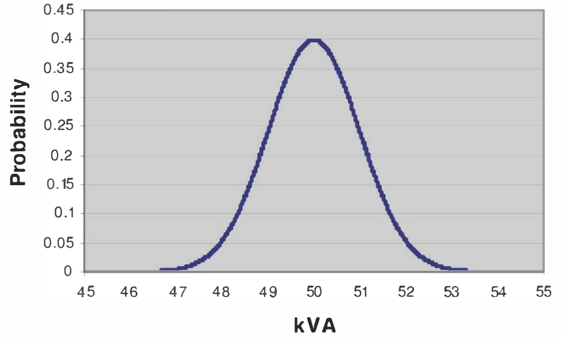

Fig. 10. Probability density function for a 50kVA distribution transformer with 1kVA standard deviation.

As previously stated, the ultimate goal is to merge the statistical nature of the predicted demand with the statistical nature of the transformer loading capability to arrive at a quantifiable statistical assessment of transformer loading and the risk associated with that loading. The probability density function, PDF, of the group peak, along with the PDF of the transformer rating can be used to determine the probability of overloading the transformer at a given group size. The PDF for the gamma distribution which describes the group peak is,

$$G(x) = f(x|\alpha,\beta) = \frac{1}{\beta^{\alpha}\Gamma(\alpha)} x^{\alpha-1} e^{\frac{-x}{\beta}} \tag{2}$$

where $\Gamma(\cdot)$ is the gamma function, and x represents demand in kVA. The input parameters, α and β, depend on the group size being observed; in general, as the group size increases, α increases.

The PDF for the transformer's rating is the normal distribution,

$$N(x) = f(x|\mu,\sigma) = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(x-\mu)^2}{2\sigma^2}} \tag{3}$$

where x is the transformer rating in kVA, σ is the standard deviation and μ is the mean of the density. An example of the PDF for the group peak as well as the transformer rating is shown in Fig. 11. The two probability density functions can be merged to determine the probability of overloading the transformer as a function of group size. This probability is given by (4).

$$Prob\_Overload = 1.0 - \int_{x=0}^{\infty} \left(G(x) * \left(\int_{x}^{\infty} N(y)dy \right) dx \right) \tag{4}$$

As an example, if the group peak, *G(x)*, for a group size of 35 (shown in Fig. 11) is to be served by a 50kVA transformer (with PDF *N(x)* shown in Fig. 11), the probability of overloading the transformer is 26.7%, as shown in Fig. 12. This figure shows the probability of overloading a 50kVA transformer for group sizes which range from 25 to 50 customers. This figure provides a direct way to assess risk in transformer loading. Using the data available to all utilities and the techniques proposed here, plots like Fig. 12 can be generated for any size transformer; and the loading assigned according to the desired amount of risk.

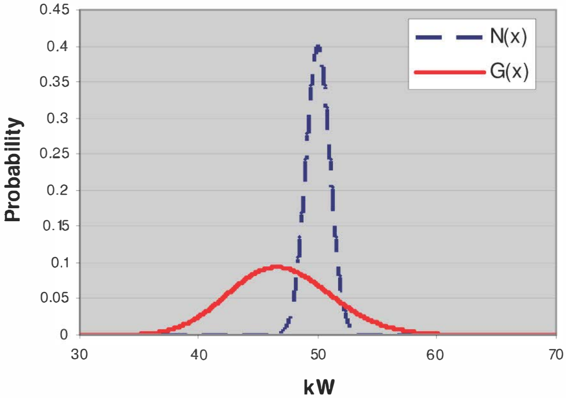

Fig. 11. Probability density function, PDF, for the group peak for 35 customers, and the PDF for a 50kVA transformer.

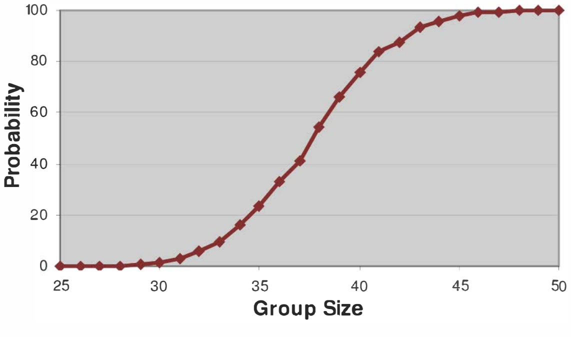

Fig. 12. Probability of overloading a 50kVA transformer as a function of group size.

VII. Conclusion

It was believed that diversity factors fit a normal distribution. This belief was based on the Chi-Square Goodness-of-Fit test and a limited number of replicates calculated by bootstrap sampling of residential loads. We show in this paper—by using the more powerful Anderson-Darling test—that diversity factors do not fit the normal distribution, but instead fit a gamma distribution. This is not surprising since both the diversity factor and the gamma distribution have a lower limit and the normal distribution has no lower bound.

We found that, to assess the risk of overloading transformers, the statistics of group peak—rather than diversity factor—are needed. Using these statistics, along with the statistical nature of the transformer loading capability, we showed how to assess risk to insulation life when assigning the number of homes to a transformer.

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IX. Biographies

Varsha P. Chatlani received the B.S.E. degree in Electrical Engineering in 2006, and is currently pursing the Ph.D. in Electrical Engineering, from Arizona State University. She is a student member of the IEEE and Eta Kappa Nu.

Daniel J. Tylavsky (SM 88) received the B.S. degree in Engineering Science in 1974, his M.S.E.E. in 1978 and the Ph.D. degree in Electrical Engineering in 1982, from the Pennsylvania State University.

From 1974-76, he worked for Basic Technology Inc. and in 1978 to 1980 he was an Instructor of Electrical Engineering at Penn State. In 1982 he joined the Faculty in the Electrical Engineering Department of Arizona State University where he is currently Associate Professor of Engineering.

He is a member of the IEEE PES and IAS societies, and is an RCA Fellow, NASA Fellow, and member of Phi Eta Sigma, Eta Kappa Nu, Tau Beta Pi, Phi Kappa Phi.

Douglas C. Montgomery is Regents' Professor of Industrial Engineering and Statistics, ASU Foundation Professor of Engineering, and Co-Director of the Graduate Program in Statistics at Arizona State University. He received the Ph.D. in engineering from Virginia Tech. His professional interests are in the development and application of statistical methodology for problems in engineering and the sciences, including new technology development, product design and commercialization, and process development.

He is a Fellow of the American Statistical Association, a Fellow of the American Society for Quality, a Fellow of the Royal Statistical Society, a Fellow of the Institute of Industrial Engineers and an Elected Member of the International Statistical Institute. He is a past editor of the *Journal of Quality Technology*, and the current editor of *Quality & Reliability Engineering International*.