Electrical Load Forecasting

Modeling and Model Construction

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To my parents, I was in need of them during my operation
To my wife, Laila, with love and respect
To my kids, Rasha, Shady, Samia, Hadeer,
and Ahmad, I love you all
To everyone who has the same liver problem,
please do not lose hope in God
(S. A. Soliman)

To my parents, who raised me
To my wife, Noureyah, with great love and respect
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for their encouragement
To my beloved family and friends
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Introduction

Economic development, throughout the world, depends directly on the availability of electric energy, especially because most industries depend almost entirely on its use. The availability of a source of continuous, cheap, and reliable energy is of foremost economic importance.

Electrical load forecasting is an important tool used to ensure that the energy supplied by utilities meets the load plus the energy lost in the system. To this end, a staff of trained personnel is needed to carry out this specialized function. Load forecasting is always defined as basically the science or art of predicting the future load on a given system, for a specified period of time ahead. These predictions may be just for a fraction of an hour ahead for operation purposes, or as much as 20 years into the future for planning purposes.

Load forecasting can be categorized into three subject areas—namely,

1. Long-range forecasting, which is used to predict loads as distant as 50 years ahead so that expansion planning can be facilitated.
2. Medium-range forecasting, which is used to predict weekly, monthly, and yearly peak loads up to 10 years ahead so that efficient operational planning can be carried out.
3. Short-range forecasting, which is used to predict loads up to a week ahead so that daily running and dispatching costs can be minimized.

In the preceding three categories, an accurate load model is required to mathematically represent the relationship between the load and influential variables such as time, weather, economic factors, etc. The precise relationship between the load and these variables is usually determined by their role in the load model. After the mathematical model is constructed, the model parameters are determined through the use of estimation techniques.

Extrapolating the mathematical relationship to the required lead time ahead and giving the corresponding values of influential variables to be available or predictable, forecasts can be made. Because factors such as weather and economic indices are increasingly difficult to predict accurately for longer lead times ahead, the greater the lead time, the less accurate the prediction is likely to be.

The final accuracy of any forecast thus depends on the load model employed, the accuracy of predicted variables, and the parameters assigned by the relevant estimation technique. Because different methods of estimation will result in different values of estimated parameters, it follows that the resulting forecasts will differ in prediction accuracy.

Over the past 50 years, the parameter estimation algorithms used in load forecasting have been limited to those based on the least error squares minimization criterion, even though estimation theory indicates that algorithms based on the least absolute value criteria are viable alternatives. Furthermore, the artificial neural network (ANN) had showed success in estimating the load for the next hour. However, the ANN used by a utility is not necessarily suitable for another utility and should be retrained to be suitable for that utility.

It is well known that the electric load is a dynamic one and does not have a precise value from one hour to another. In this book, fuzzy systems theory is implemented to estimate the load model parameters, which are assumed to be fuzzy parameters having a certain middle and spread. Different membership functions, for load parameters, are used—namely, triangular membership and trapezoidal membership functions. The problem of load forecasting in this book is restricted to short-term load forecasting and is formulated as a linear estimation problem in the parameters to be estimated. In this book, the parameters in the first part are assumed to be crisp parameters, whereas in the rest of the book these parameters are assumed to be fuzzy parameters. The objective is to minimize the spread of the available data points, taking into consideration the type of membership of the fuzzy parameters, subject to satisfying constraints on each measurement point, to ensure that the original membership is included in the estimated membership.

Outline of the Book

In this book, different techniques used in the past two decades are implemented to estimate the load model parameters, including fuzzy parameters with certain middle and certain spread. The book contains nine chapters:

Chapter 1, "Mathematical Background and State of the Art." This chapter introduces mathematical background to help the reader understand the problems formulated in this book. In this chapter, the reader will study matrices and their applications in estimation theory and see that the use of matrix notation simplifies complex mathematical expressions. The simplifying matrix notation may not reduce the amount of work required to solve mathematical equations, but it usually makes the equations much easier to handle and manipulate. This chapter explains the vectors and the formulation of quadratic forms, and, as we shall see, that most objective functions to be minimized (least errors square criteria) are quadratic in nature. This chapter also explains some optimization techniques and introduces the concept of a state space model, which is commonly used in dynamic state estimation. The reader will also review different techniques that, developed for the short term, give the state of the art of the various algorithms used during the past decades for short-term load forecasting. A brief discussion for each algorithm is presented in this chapter. Advantages and disadvantages of each algorithm are discussed. Reviewing the most recent publications in the area of short-term load forecasting indicates that most of the available algorithms treat the parameters of the proposed load model as crisp parameters, which is not the case in reality.

Chapter 2, "Static State Estimation." This chapter presents the theory involved in different approaches that use parameter estimation algorithms. In the first part of the chapter, the crisp parameter estimation algorithms are presented; they include the least error squares (LES) algorithm and the least absolute value (LAV) algorithm. The second part of the chapter presents an introduction to fuzzy set theory and systems, followed by a discussion of fuzzy linear regression algorithms. Different cases for the fuzzy parameters are discussed in this part. The first case is for the fuzzy linear regression of the linear models having fuzzy parameters with nonfuzzy outputs, the second case is for the linear regression of fuzzy parameters with fuzzy output, and the third case is for fuzzy parameters formulated with fuzzy output of left and right type (LR-type).

Chapter 3, "Load Modeling for Short-Term Forecasting." This chapter proposes different load models used in short-term load forecasting for 24 hours.

- Three models are proposed in this chapter—namely, models A, B, and C. Model A is a multiple linear regression model of the temperature deviation, base load, and either wind-chill factor for winter load or temperature humidity factor for summer load. The parameters of load A are assumed to be crisp parameters in this chapter. The term *crisp parameters* mean clearly defined parameter values without ambiguity.
- Load model B is a harmonic decomposition model that expresses the load at any instant, t, as a harmonic series. In this model, the weekly cycle is accounted for through use of a daily load model, the parameters of which are estimated seven times weekly. Again, the parameters of this model are assumed to be crisp.
- Load model C is a hybrid load model that expresses the load as the sum of a time-varying base load and a weather-dependent component. This model is developed with the aim of eliminating the disadvantages of the other two models by combining their modeling approaches. After finding the parameter values, one uses them to determine the electric load from which these parameter values are extracted, and this value is called the estimated load. Then the parameter values are used to predict the electric load for a randomly chosen day in the future, and it is called the predicted load for that chosen day.

Chapter 4, "Fuzzy Regression Systems and Fuzzy Linear Models." The objective of this chapter is to introduce principal concepts and mathematical notions of fuzzy set theory, a theory of classes of objects with non sharp boundaries.

- We first review fuzzy sets as a generalization of classical crisp sets by extending the range of the membership function (or characteristic function) from [0, 1] to all real numbers in the interval [0, 1].
- A number of notions of fuzzy sets, such as representation support, α-cuts, convexity, and fuzzy numbers, are then introduced. The resolution principle, which can be used to expand a fuzzy set in terms of its α-cuts, is discussed.
- This chapter introduces fuzzy mathematical programming and fuzzy multiple-objective decision making. We first introduce the required knowledge of fuzzy set theory and fuzzy mathematics in this chapter.
- Fuzzy linear regression also is introduced in this chapter; the first part is to estimate the fuzzy regression coefficients when the set of measurements available is crisp, whereas in the second part the fuzzy regression coefficients are estimated when the available set of measurements is a fuzzy set with a certain middle and spread.
- Some simple examples for fuzzy linear regression are introduced in this chapter.

- The models proposed in Chapter 3 for crisp parameters are used in this chapter. Fuzzy model A employs a multiple fuzzy linear regression model. The membership function for the model parameters is developed, where triangular membership functions are assumed for each parameter of the load model. Two constraints are imposed on each load measurement to ensure that the original membership is included in the estimated membership.
- Fuzzy model B, which is a harmonic model, also is proposed in this chapter. This model involves fuzzy parameters having a certain median and certain spread.
- Finally, a hybrid fuzzy model C, which is the combination of the multiple linear regression model A and harmonic model B, is presented in this chapter.

Chapter 5, "Dynamic State Estimation." The objective of this chapter is to study the dynamic state estimation problem and its applications to electric power system analysis, especially short-term load forecasting. Furthermore, the different approaches used to solve this dynamic estimation problem are also discussed in this chapter. After reading this chapter, the reader will be familiar with

The five fundamental components of an estimation problem:
- The variables to be estimated.
- The measurements or observations available.
- The mathematical model describing how the measurements are related to the variable of interest.
- The mathematical model of the uncertainties present.
- The performance evaluation criterion to judge which estimation algorithms are "best."

Formulation of the dynamic state estimation problem:
- Kalman filtering algorithm as a recursive filter used to solve a problem.
- Weighted least absolute value filter.
- Different problems that face Kalman filtering and weighted least absolute value filtering algorithms.

Chapter 6, "Load Forecasting Results Using Static State Estimation." The objective of this chapter is as follows:

In Chapter 3, the models are derived on the basis that the load powers are crisp in nature; the data available from a big company in Canada are used to forecast the load power in the crisp case.
- In this chapter, the results obtained for the crisp load power data for the different load models developed in Chapter 3 are shown.
- A comparison is performed between the two static LES and LAV estimation techniques.
- The parameters estimated are used to predict a load using both techniques, where we compare between them for summer and winter.

Chapter 7, "Load Forecasting Results Using Fuzzy Systems." Chapter 6 discusses the short-term load-forecasting problem, and the LES and LAV parameter estimation algorithms are used to estimate the load model parameters. The error in the estimates is calculated for both techniques. The three models, proposed earlier in Chapter 3, are used in that chapter to present the load in different days for different seasons. In this chapter, the fuzzy load models developed in Chapter 5 are tested. The fuzzy parameters of these models are estimated using the past history data for summer weekdays and weekend days as well as for winter weekdays and weekend days. Then these models are used to predict the fuzzy load power for 24 hours ahead, in both summer and winter seasons. The results are given in the form of tables and figures for the estimated and predicted loads.

Chapter 8, **"Dynamic Electric Load Forecasting."** The main objectives of this chapter are as follows:

- A one-year long-term electric power load-forecasting problem is introduced as a first step for short-term load forecasting.
- A dynamic algorithm, the Kalman filtering algorithm, is suitable to forecast daily load profiles with a lead-time from several weeks to a few years.
- The algorithm is based mainly on multiple simple linear regression models used to capture the shape of the load over a certain period of time (one year) in a two-dimensional layout (24 hours $\times$ 52 weeks).
- The regression models are recursively used to project the 2D load shape for the next period of time (next year). Load-demand annual growth is estimated and incorporated into the Kalman filtering algorithm to improve the load-forecast accuracy obtained so far from the regression models.

Chapter 9, **"Electric Load Modeling for Long-Term Forecasting."** The objectives of this chapter are as follows:

- This chapter provides a comparative study between two static estimation algorithms—namely, the least error squares (LES) and least absolute value (LAV) algorithms—for estimating the parameters of different load models for peak-load forecasting necessary for long-term power system planning.
- The proposed algorithms use the past history data for the load and the influence factors, such as gross domestic product (GDP), population, GDP per capita, system losses, load factor, etc.
- The problem turns out to be a linear estimation problem in the load parameters. Different models are developed and discussed in the text.

4 Fuzzy Regression Systems and Fuzzy Linear Models

4.1 Objectives

The objectives of this chapter are

- Introducing principal concepts and mathematical notions of fuzzy set theory, a theory of classes of objects with nonsharp boundaries.
- Reviewing fuzzy sets as a generalization of classical crisp sets by extending the range of the membership function (or characteristic function) from [0, 1] to all real numbers in the interval [0, 1].
- Introducing a number of notions of fuzzy sets, such as representation support, α-cuts, convexity, and fuzzy numbers. We also discuss the resolution principle, which can be used to expand a fuzzy set in terms of its α-cuts.
- Introducing fuzzy mathematical programming and fuzzy multiple-objective decision making. We first introduce the required knowledge behind fuzzy set theory and fuzzy mathematics.
- Introducing fuzzy linear regression. The first part of this discussion describes how to estimate the fuzzy regression coefficients when the set of measurements available is crisp, whereas in the second part the fuzzy regression coefficients are estimated when the available set of measurements is a fuzzy set with a certain middle and spread.
- Introducing some simple examples for fuzzy linear regression.

4.2 Fuzzy Fundamentals

Human beings make tools for their use and also want to control the tools as they desire. A feedback concept is very important in being able to achieve control over these tools. As modern plants with many inputs and outputs become more and more complex, any description of a modern control system requires a large number of equations. Since about 1960, modern control theory has been developed to cope with the increased complexity of modern plants. The most recent developments may be said to be in the direction of optimal control of both deterministic and stochastic systems, as well as the adaptive and learning control of time-variant complex systems. These developments have been accelerated through the use of digital computers.

Modern plants are designed for efficient analysis and production by human beings. We are now confronted with the need to control living cells, which are nonlinear, complex, time variant, and mysterious. They cannot be mastered easily through classical or control theory or even modern artificial intelligence (AI) employing a

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powerful digital computer. So we are faced with many problems, and our problems can be seen in terms of decisions, management, and predictions. Solutions can be seen in terms of faster access to more information and of increased aid in analyzing, understanding, and utilizing the information that is not available. These two elements, a large amount of information coupled with a large amount of uncertainty, taken together constitute the basis for many of our problems today: complexity. How do we manage to cope with complexity as well as we do, and how could we manage to cope better? These are the reasons for introducing fuzzy notations because the fuzzy sets method is very useful for handling uncertainties and is essential for the knowledge acquisition of human experts. First, we have to know what *fuzzy* means? Fuzzy essentially means vague or imprecise information.

Everyday language provides one example of the way vagueness is used and propagated; for example, consider driving a car or describing the weather or classifying a person's age. So using the term *fuzzy* is one way engineers describe the operation of a system by means of fuzzy variables and terms. To solve any control problem, we might have a variable. This variable is a crisp set in the conventional control method; that is, it has a definite value and a certain boundary in such a way it can be defined by two groups:

1. Members, or those that certainly belong in the set inside the boundary.
2. Nonmembers, or those that certainly don't belong.

But sometimes collections and categories have boundaries that seem vague, and the transition from member to nonmember appears gradual rather than abrupt. These collections and categories are what we call *fuzzy sets*. Thus, fuzzy sets are a generalization of conventional set theory. Every fuzzy set can be represented by a membership function, and there is no unique membership. A function for any fuzzy set, a membership function, exhibits a continuous curve changing from 0 to 1 or vice versa, and this transition region represents a fuzzy boundary of the term.

For a computer language, we can define *fuzzy logic* as a method of easily representing analog processes with continuous phenomena that are not easily broken down into discrete segments, and the concepts involved are difficult to model sometimes. In conclusion, we can use the term *fuzzy* when

1. One or more of the control variables are continuous.
2. A mathematical model of the process does not exist, or it exists but is too difficult to encode.
3. A mathematical model is too complex to be evaluated fast enough for real-time operation.
4. A mathematical model involves too much memory on the designated chip architecture.
5. An expert is available who can specify the rules underlying the system behavior and the fuzzy sets that represent the characteristics of each variable.
6. A system has uncertainties in either its inputs or definition.

On the other hand, for systems in which conventional control equations and methods are already optimal or entirely adequate, we should avoid using fuzzy logic. One of the advantages of fuzzy logic is that we can implement systems too complex, too nonlinear, or with too much uncertainty to implement using traditional techniques. We also can implement and modify systems more quickly and squeeze additional capability from existing designs. Finally, fuzzy logic is simple to describe and verify. Before we introduce fuzzy models, however, we need to know some definitions:

- **Singleton:** A deterministic word of term or value (e.g., *male* or *female*, *dead* or *alive*, *80°C*, *30 Kg*). These deterministic words and numerical values have neither flexibility nor intervals. So a numerical value to be substituted into a mathematical equation representing a scientific law is a *singleton*.
- **Fuzzy number:** A fuzzy linguistic term that includes imprecise numerical value (e.g., "around 80°C," "bigger than 25").
- **Fuzzy set:** A fuzzy linguistic term that can be regarded as a set of singletons; the grades of it are not only [1] but also range from zero to one [0, 1]. Alternatively, it is a set that allows partial membership states. Whether ordinary or crisp, sets have only two membership states: inclusion and exclusion (member and nonmember). Fuzzy sets allow a degree of membership as well. Fuzzy sets are defined by labels and membership functions, and every fuzzy set has an infinite number of membership functions (μFs) that may represent it.
- **Fuzzy linguistic terms:** Elements that are ordered are fuzzy intervals, and the membership function is a bandwidth of this fuzzy linguistic term. Elements of fuzzy linguistic terms such as "robust gentleman" and "beautiful lady" are discrete and also disordered. This type of term cannot be defined by a continuous membership function, but defined by vectors.
- **Characteristic function:** This is comprised of a singleton, an interval, and a fuzzy linguistic term.
- **Control variable:** A variable that appears in the premise of a rule and controls the state of the solution variables.
- **Defuzzification:** The process of converting an output fuzzy set for a solution variable into a single value that can be used as output.
- **Overlap:** The degree to which the domain of one fuzzy set overlaps with that of another.
- **Solution fuzzy set:** A temporary fuzzy set created by the fuzzy model to resolve the value of a corresponding solution variable. When all the rules have been fired, the solution fuzzy set is defuzzified into the actual solution variable.
- **Solution variable:** The variable of which the value the fuzzy logic system is meant to find.
- **Fuzzy model:** The components of conventional and fuzzy systems are quite alike, differing mainly in that fuzzy systems contain "fuzzifers," which convert inputs into their fuzzy representations, and "defuzzifiers," which convert the output of the fuzzy process logic into "crisp" (numerically precise) solution variables.

In a fuzzy system, the values of a fuzzified input execute all the values in the knowledge repository that have the fuzzified input as part of the premise. This process generates a new fuzzy set representing each output or solution variable. Defuzzification creates a value for the output variable from that new fuzzy set. For physical systems, the output value is often used to adjust the setting of an actuator that, in turn, adjusts the states of the physical systems. The change is picked up by the sensors, and the entire process starts again. Finally, we can say that there are four steps to follow to design a fuzzy model.

Step 1 Define the Model Function and Operational Characteristics

The goal of the first step in designing a fuzzy model is to establish the architectural characteristics of a system and also to define the specific operating properties of the proposed fuzzy system. The fuzzy system designer's task lies in defining what information (data point) flows into the system, what basic operations are performed on the data, and what data elements are output from the system. Even if lacking a mathematical model of the system process, the designer must have a deep understanding of these three phenomena. This step is also the time to define exactly where the fuzzy subsystem fits into the total system architecture, which provides a clear picture of how inputs and outputs flow to and from the subsystem. Then the designer can estimate the number and ranges of inputs and outputs that will be required. This step also reinforces the input process–output design step.

Step 2 Define the Control Surfaces

Each control and solution variable in the fuzzy model is decomposed into a set of fuzzy regions. These regions are given a unique name, called labels, within the domain of the variable. Finally, a fuzzy set that semantically represents the concept associated with the label is created. Some rules of thumb help in defining fuzzy sets:

- First, the number of labels associated with a variable should generally be an odd number from 5 to 9.
- Second, each label should overlap somewhat with its neighbors. To get a smooth stable surface fuzzy controller, the overlap should be between 10% and 50% of the neighboring space, and the sum of vertical points of the overlap should always be less than one.
- Third, the density of the fuzzy sets should be highest around the optimal control point of the system and should decrease as the distance from that point increases.

Step 3 Define the Behavior of the Control Surfaces

The third step in designing a fuzzy model involves writing the rules that tie the input values to the output model properties. These rules are expressed in English-like language with syntax like the following:

If <*fuzzy proposition*>, then <*fuzzy proposition*>

That is, the IF, THEN rule, where fuzzy propositions are "*x* is *y*" or "*x* is not *y*." *x* is a scalar variable, and *y* is a fuzzy set associated with that variable. Generally, the number of rules a system requires is simply related to the number of control variables.

Step 4 Select a Method of Defuzzification

The fourth step in designing a fuzzy model is finding a way to convert an output fuzzy set into a crisp solution variable. The two most common ways are

- The composite maximum
- The composite momentary cancroids

Once the fuzzy model has been constructed, the process of solution and protocycling begins. The model is compared against known test cases to validate the results. When the results are not as desired, changes are made either to the fuzzy set descriptions or to the mappings encoded in the rules.

4.3 Fuzzy Sets and Membership

Fuzzy set theory is developed to improve the oversimplified model, thereby developing a more robust and flexible model to solve real-world complex systems involving human aspects [1,2]. Furthermore, it helps the decision maker not only to consider the existing alternatives under given constraints (optimize a given system), but also to develop new alternatives (design a system). Fuzzy set theory has been applied in many fields, such as operations research, management science, control theory, artificial intelligence/expert system, human behavior, etc.

4.3.1 Membership Functions

A classical (crisp or hard) set is a collection of distinct objects, defined in such a manner as to separate the elements of a given universe of discourse into two groups: those that belong (members) and those that do not belong (nonmembers). The transition of an element between membership and nonmembership in a given set in the universe is abrupt and well defined. The crisp set can be defined by the so-called characteristic function. Let U be a universe of discourse, the characteristic function of a crisp.

4.3.2 Basic Terminology and Definitions

Let X be a classical set of objects, called the universe, of which the generic elements are denoted by x [2]. The membership in a crisp subset of X is often viewed as a characteristic function μ_A from X to $\{0, 1\}$ such that

$$\begin{aligned} \mu_A(x) &= 1 \quad \text{if and only if } x \in \mathrm{A} \\ &= 0 \quad \text{otherwise} \end{aligned} \tag{4.1}$$

where $\{0, 1\}$ is called a valuation set.

If the valuation set is allowed to be the real interval [0, 1], $\widetilde{A}$ is called a fuzzy set proposed by Zadeh [2], and $\mu_A(x)$ is the degree of membership of x in $\widetilde{A}$. The closer the value of $\mu_A(x)$ is to 1, the more x belongs to $\widetilde{A}$ [2]. Therefore, $\widetilde{A}$ is completely characterized by the set of ordered pairs:

$$\widetilde{A} = \{(x, \mu_A(x)) \mid x \in X\} \tag{4.2}$$

It is worth noting that the characteristic function can be either a membership function or a possibility distribution. In this study, if the membership function is preferred, then the characteristic function will be denoted as $\mu_A(x)$. On the other hand, if the possibility distribution is preferred, the characteristic function will be specified as $\pi(x)$. Along with the expression of equation (4.2), Zadeh [2] also proposed the following notations. When X is a finite set $\{x_1, x_2, \ldots, x_n\}$, a fuzzy set $\widetilde{A}$ is then expressed as

$$\widetilde{A} = \mu_A(x_1)/x_1 + \cdots + \mu_A(x_n)/x_n = \sum_i \mu_A(x_i)/x_i \tag{4.3}$$

When X is not a finite set, A then can be written as

$$A = \int_X \mu_A(x)/x \tag{4.4}$$

Sometimes, we might need only objects of a fuzzy set but not its characteristic function to transfer a fuzzy set. To do so, we must consider two concepts: support and α-level cut.

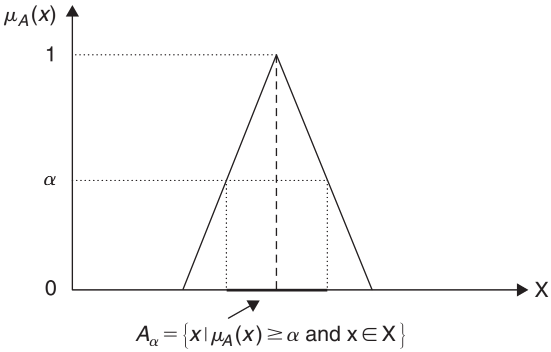


Figure 4.1 The α-level set (α-cut) of a fuzzy set A.

4.3.3 Support of a Fuzzy Set

The support of a fuzzy set A is the crisp set of all $x \in U$ such that $(x) > 0$ [1,2]. That is,

$$\mathrm{supp}(A) = \{x \in U | \mu_A > 0\} \tag{4.5}$$

The α-level set (α-cut) of a fuzzy set A is a crisp subset of X and is denoted by Figure 4.1. An α-cut of a fuzzy set $\widetilde{A}$ is a crisp set A, which contains all the elements of the universe U that have a membership grade in $\widetilde{A}$ greater than or equal to α. That is,

$$A_\alpha = \{x | \mu_A(x) \geq \alpha \,\text{and}\, x \in X\} \tag{4.6}$$

If $A_\alpha = \{x | \mu_A(x) > \alpha\}$, then A_α is called a strong α-cut of a given fuzzy set A or is called a level set of A. That is,

$$\prod\nolimits_A = \{\alpha | \mu_A(x) = \alpha, \text{ for some } x \in \mathsf{U}\} \tag{4.7}$$

4.3.4 Normality

A fuzzy set A is normal if and only if $\mathrm{Sup}_x \, \mu_A(x) = 1$; that is, the supreme of $\mu_A(x)$ over X is unity. A fuzzy set is subnormal if it is not normal. A nonempty subnormal fuzzy set can be

$$A_\alpha = \{x | \mu_A(x) \geq \alpha \,\text{and}\, x \in X\}$$

normalized by dividing each $\mu_A(x)$ by the factor $\mathrm{Sup}_x \mu_A(x)$. A fuzzy set is empty if and only if $\mu_A(x) = 0 \,\text{for}\, \forall x \in X) \forall x \in X$.

4.3.5 Convexity and Concavity

A fuzzy set A in X is convex if and only if for every pair of point x^1 and x^2 in X, the membership function of A satisfies the inequality

$$\mu_A(\partial x^1 + (1 - \partial)x^2) \geq \min(\mu_A(x^1), \mu_A(x^2)) \tag{4.8}$$

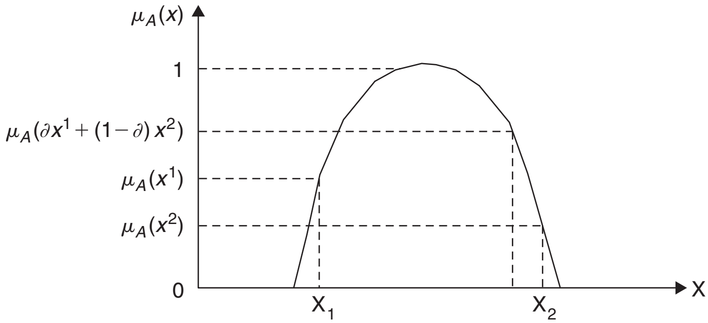


Figure 4.2 A convex fuzzy set.

where $\partial \in [0,1]$ (see Figure 4.2). Alternatively, a fuzzy set is convex if all α-level sets are convex.

Dually, A is concave if its complement A^c is convex. It is easy to show that if A and B are convex, so is $A \cap B$. Dually, if A and B are concave, so is $A \cup B$.

4.3.6 Basic Operation

This section provides a summary of some basic set-theoretic operations that are useful in fuzzy mathematical programming and fuzzy multiple-objective decision making. These operations are based on the definitions from Bellman and Zadeh [1].

1. **Inclusion**
   Let A and B be two fuzzy subsets of X. Then A is included in B if and only if

$$A_\alpha = \{x|\mu_A(x) \geq \alpha \text{ and } x \in X\}$$
$$\mu_A(x) \leq \mu_B(x) \text{ for } \forall x \in X \tag{4.9}$$

2. **Equality**
   A and B are called equal if and only if

$$\mu_A(x) = \mu_B(x) \text{ for } \forall x \in X \tag{4.10}$$

3. **Complementation**
   A and B are complementary if and only if

$$\mu_A(x) = 1 - \mu_B(x) \text{ for } \forall x \in X \tag{4.11}$$

4. **Intersection**
   The intersection of A and B may be denoted by $A \cap B$, which is the largest fuzzy subset contained in both fuzzy subsets A and B. When the min operator is used to express the logical "and," its corresponding membership is then characterized by

$$\begin{aligned}\mu_{A\cap B}(x) &= \min(\mu_A(x), \mu_B(x)) \text{ for } \forall x \in X \\ &= \mu_A(x) \wedge \mu_B(x)\end{aligned} \tag{4.12}$$

   where $\wedge$ is a conjunction.

5. Union

The union ($A \cup B$) of A and B is dual to the notion of intersection. Thus, the union of A and B is defined as the smallest fuzzy set containing both A and B.

The membership function of A ∪ B is given by

$$\begin{aligned} \mu_{A \cup B}(x) &= \max(\mu_A(x), \mu_B(x)) \text{ for } \forall x \in X \\ &= \mu_A(x) \vee \mu_B(x) \end{aligned} \tag{4.13}$$

6. Algebraic Product

The algebraic product AB of A and B is characterized by the following membership function:

$$\mu_{A \cup B}(x) = \mu_A(x)\, \mu_B(x) \text{ for } \forall x \in X \tag{4.14}$$

7. Algebraic Sum

The algebraic sum $A \oplus B$ of A and B is characterized by the following membership function:

$$\mu_{A \oplus B}(x) = \mu_A(x) + \mu_B(x) - \mu_A(x)\mu_B(x) \tag{4.15}$$

8. Difference

The difference $A - B$ of A and B is characterized by

$$\mu_{A \cap B^c}(x) = \min(\mu_A(x), \mu_{B^c}(x)) \tag{4.16}$$

9. Fuzzy Arithmetic

a. Addition of Fuzzy Numbers

The addition of X and Y can be calculated by using α-level cut and max-min convolution.

α-level cut. Using the concept of confidence intervals, the α-level sets of X and Y are $X_\alpha = \left[X_\alpha^L, X_\alpha^U\right]$ and $Y_\alpha = \left[Y_\alpha^L, Y_\alpha^U\right]$, where the result, Z, of the addition is

$$Z_\alpha = X_\alpha(+)Y_\alpha = \left[X_\alpha^L + Y_\alpha^L, X_\alpha^U + Y_\alpha^U\right] \tag{4.17}$$

for every $\alpha \in [0, 1]$.

Max-min convolution. The addition of the fuzzy numbers X and Y is represented as

$$Z(z) = \max_{z=x+y} \left[\min[\mu_X(x), \mu_Y(y)]\right] \tag{4.18}$$

b. Subtraction of Fuzzy Numbers

α-level cut. The subtraction of the fuzzy numbers X and Y in the α-level cut representation is

$$Z_\alpha = X_\alpha(-)Y_\alpha = \left[X_\alpha^L - Y_\alpha^U, X_\alpha^U - Y_\alpha^L\right] \tag{4.19}$$

for every $\alpha \in [0,1]$.

Max-min convolution. The subtraction of the fuzzy numbers X and Y is represented as

$$\begin{aligned} \mu_Z(Z) = &\max_{z=x-y} \{[\mu_x(x), \mu_Y(y)]\} \\ &\max_{z=x+y} \{[\mu_x(x), \mu_Y(-y)]\} \\ &\max_{z=x+y} \{[\mu_x(x), \mu_{-Y}(y)]\} \end{aligned} \tag{4.20}$$

c. Multiplication of Fuzzy Numbers

***α*-level cut**. The multiplication of the fuzzy numbers X and Y in the α-level cut representation is

$$Z_\alpha = X_\alpha(.)Y_\alpha = \left[X_\alpha^L y_\alpha^L, X_\alpha^U Y_\alpha^U\right] \tag{4.21}$$

for every $\alpha \in [0,1]$.

Max-min convolution. The multiplication of the fuzzy numbers X and Y is represented by Kaufmann and Gupta [2] in the following procedure as

1. Find Z^1 (the peak of the fuzzy number Z) such that $\mu_Z(z^1) = 1$; then calculate the left and right legs.
2. The left leg of $\mu_Z(z)$ is defined as

$$\mu_z(z) = \max_{xy \leq z}\{\min[\mu_x(x), \mu_Y(y)]\} \tag{4.22}$$

3. The right leg of $\mu_Z(z)$ is defined as

$$\mu_z(z) = \max_{xy \geq z}\{\min[\mu_x(x), \mu_Y(y)]\} \tag{4.23}$$

d. Division of Fuzzy Numbers

***α*-level cut**. The division is represented as follows:

$$Z_\alpha = X_\alpha(:)Y_\alpha = \left[x_\alpha^L/y_\alpha^U, x_\alpha^U/y_\alpha^L\right] \tag{4.24}$$

Max-min convolution. As defined earlier, we must find the peak and then the left and right legs:

1. The peak $Z = X$ (:) Y is used.
2. The left leg is presented as

$$\begin{aligned}\mu_z(z) = & \max_{x/y \leq z}\{\min[\mu_x(x), \mu_Y(y)]\} \\ & \max_{xy \leq z}\{\min[\mu_x(x), \mu_Y(1/y)]\} \\ & \max_{xy \leq z}\{\min[\mu_x(x), \mu_{1/Y}(y)]\}\end{aligned} \tag{4.25}$$

3. The right leg is presented as

$$\begin{aligned}\mu_z(z) = & \max_{x/y \geq z}\{\min[\mu_x(x), \mu_Y(y)]\} \\ & \max_{xy \geq z}\{\min[\mu_x(x), \mu_Y(1/y)]\} \\ & \max_{xy \geq z}\{\min[\mu_x(x), \mu_{1/Y}(y)]\}\end{aligned} \tag{4.26}$$

10. *LR*-Type Fuzzy Number

A fuzzy number is defined to be of the *LR* type if there are reference functions L and R and positive scalars, as shown in Figure 4.3, α (left spread), β (right spread), and m (mean), such that

$$\mu_M(x) = \left\{\begin{array}{l} L\left(\dfrac{m-x}{\alpha}\right) \text{for } x \leq m \\ R\left(\dfrac{x-m}{\beta}\right) \text{for } x \geq m \end{array}\right\} \tag{4.27}$$

As the spread increases, M becomes fuzzier and fuzzier. Symbolically, we write

$$M = (m,\ \alpha\beta)_{LR} \tag{4.28}$$

11. **Interval Arithmetic**
Interval arithmetic is normally used with uncertain data obtained from different instruments. If we enclose those values obtained in a closed interval on the real line R—that is, this uncertain value is inside an interval of confidence R—$x \in [a_1,\ a_2]$, where $a_1 \leq a_2$.
12. **Triangular and Trapezoidal Fuzzy Numbers**
Triangular and trapezoidal fuzzy numbers are considered among the most important and useful tools in solving possibility mathematical programming problems. Tables 4.1 and 4.2 show all the formulas used in the LR representation of fuzzy numbers and interval arithmetic methods.

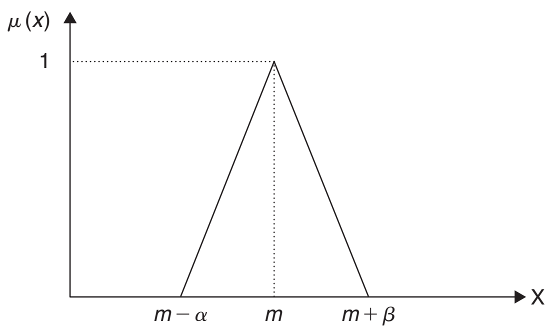


Figure 4.3 *LR*-type fuzzy number.

Table 4.1 Fuzzy Arithmetic on Triangular *LR* Representation of Fuzzy Numbers; $X = (x, \alpha, \beta)$ & $Y = (y, r, \delta)$

Image of Y:	$-Y = (-y, \delta, r)\ -Y = (-y, \delta, r)$
Inverse of Y:	$Y^{-1} = (y^{-1}, \delta y^{-2}, r y^{-2})$
Addition:	$X\ (+)\ Y = (x + y, \alpha + r, \beta + \delta)$
Subtraction:	$X\ (-)\ Y = X(+) - Y = (x - y, \alpha + \delta, \beta + r)$
Multiplication:	$X > 0,\ Y > 0 : X\ (\cdot)\ Y = (xy, xr + y\alpha, x\delta + y\beta)$
	$X < 0,\ Y > 0 : X\ (\cdot)\ Y = (xy, y\alpha - x\delta, y\beta - xr)$
	$X < 0,\ Y < 0 : X\ (\cdot)\ Y = (xy,\ -x\delta - y\beta,\ -xr - y\alpha)$
Scalar Multiplication:	$a > 0, a \in R : a\ (\cdot)\ X = (ax, a\alpha, a\beta)$
	$a < 0, a \in R : a\ (\cdot)\ X = (ax,\ -a\beta,\ -a\alpha)$
Division:	$X > 0,\ Y > 0 : X\ (:)\ Y = (x/y, (x\delta + y\alpha)/y^2, (xr + y\beta)/y^2)$
	$X < 0,\ Y > 0 : X\ (:)\ Y = (x/y, (y\alpha - xr)/y^2, (y\beta - x\delta)/y^2)$
	$X < 0,\ Y < 0 : X\ (:)\ Y = (x/y, (-xr - y\beta)/y^2, (-x\delta - y\alpha)/y^2)$

Table 4.2 Fuzzy Interval Arithmetic on Triangular Fuzzy Numbers; $X = (x^m, x^p, x^o)$ & $Y = (y^m, y^p, y^o)$

Image of Y:	$-Y = (-y^m, -y^o, -y^p)$
Inverse of Y:	$Y^{-1} = (1/y^m, 1/y^o, 1/y^p)$
Addition:	$X\ (+)\ Y = (x^m + y^m, x^p + y^p, x^o + y^o)$
Subtraction:	$X\ (-)\ Y = X(+) - Y = (x^m - y^m, x^p - y^o, x^o - y^p)$
Multiplication:	$X > 0,\ Y > 0 : X\ (\cdot)\ Y = (x^m y^m, x^p y^p, x^o y^o)$
	$X < 0,\ Y > 0 : X\ (\cdot)\ Y = (x^m y^m, x^p y^o, x^o y^p)$
	$X < 0,\ Y < 0 : X\ (\cdot)\ Y = (x^m y^m, x^o y^o, x^p y^p)$
Scalar Multiplication:	$a > 0, a \in R : a(\cdot)X = (ax^m, ax^p, ax^o)$
	$a < 0, a \in R : a(\cdot)X = (ax^m, ax^o, ax^p)$
Division:	$X > 0,\ Y > 0 : X\ (:)\ Y = (x^m/y^m, x^p/y^o, x^o/y^p)$
	$X < 0,\ Y > 0 : X\ (:)\ Y = (x^m/y^m, x^o/y^o, x^p/y^p)$
	$X < 0,\ Y < 0 : X\ (:)\ Y = (x^m/y^m, x^o/y^p, x^p/y^o)$

4.4 Fuzzy Linear Estimation

The fuzzy parameters linear estimation model or fuzzy regression model can be described by the following equation [3–13]:

$$Y = f(x, A) = A_1 x_1 + A_2 x_2 + A_3 x_3 + \cdots + A_n x_n \tag{4.29}$$

At any observation j; $j = 1, 2, \ldots, m$, equation (4.29) can be rewritten as

$$Y_j = f(x, A) = A_1 x_{1j} + A_2 x_{2j} + A_3 x_{3j} + \cdots + A_n x_{nj} \tag{4.30}$$

In fuzzy regression, the difference between the observed and estimated values is assumed to be due to the ambiguity inherently present in the system. Therefore, the preceding fuzzy regression model is built in terms of the possibility and evaluates all observed values as possibilities that the system should contain. The model in equation (4.29) is named as a possible regression model. In this model Y_j is the observation measurement j. This output observation may be a nonfuzzy or fuzzy observation; A_i, $i = 1, 2, \ldots, n$ are the fuzzy parameters of the model in the form of (p_i, c_i), where p_i is the middle and c_i is the spread. Or, it may take the form of the LR type as (p_i, c_i^L, c_i^R), and x_{ij} is the input to the model $i = 1, 2, \ldots, n$ and $j = 1, 2, \ldots, m$. In this section, three cases for the output Y_j are studied.

4.4.1 Nonfuzzy Output ($Y_j = m_j$)

In the nonfuzzy output model, the output Y_j is a nonfuzzy observation, but the model coefficients A_i, $i = 1, 2, \ldots, n$ are fuzzy parameters either in the form of $A_i = (p_i, c_i)$ or, $A_i = \left(p_i, c_i^L, c_i^R\right)$, $i = 1, \ldots, n$ for the LR type and the input x_{ij} is a nonfuzzy input. The membership functions for each type of A_i are shown in Figures 4.4 and 4.5.

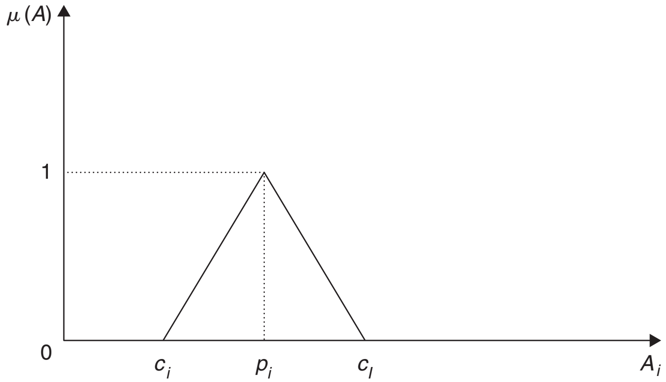


Figure 4.4 Membership functions of the fuzzy coefficients A_j.

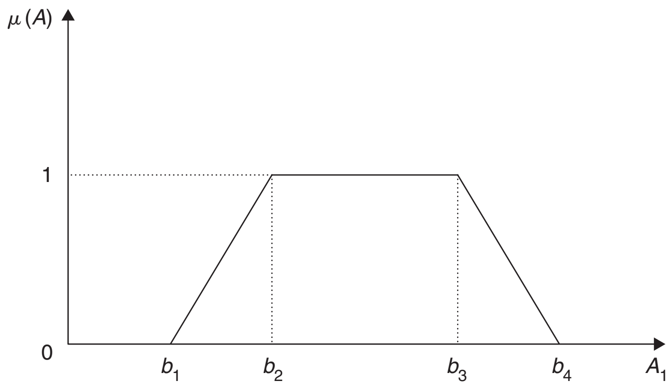


Figure 4.5 Trapezoidal membership function of A_j.

The equation that describes this membership can be written mathematically, for the triangular fuzzy number, as

$$\mu_A = \begin{cases} 1 - \dfrac{|p_j - a_i|}{c_j}; & p_j - c_j \le a_{i \le} p_j + c_j \\ 0 & \text{otherwise} \end{cases} \tag{4.31}$$

where the membership function of A_j of the LR type is assumed to be a trapezoidal function, as shown in Figure 4.5. Note that if $b_2 = b_3$, we obtain the triangular membership. In general, the membership function for the LR type can be described as

$$\mu_A = \begin{cases} L\left(p_j - \dfrac{x}{c_j^L}\right) & \text{for } x \le p_j \\ R\left(p_j - \dfrac{x}{c_j^R}\right) & \text{for } x \ge p_j \end{cases} \tag{4.32}$$

where p_i is the middle or the mean of A_j, c_j^L is the left spread, and c_i^R is the right spread.

Equation (4.29) can now be written as

$$Y_j = (p_1, c_1)x_{1j} + (p_2, c_2)x_{2j} + (p_n, c_n)x_{nj}, \qquad j = 1, 2, \ldots, m \tag{4.33}$$

for the first type of the fuzzy parameters and

$$Y_j = (p_1, c_1^L, c_1^R)x_{1j} + (p_2, c_2^L, c_2^R)x_{2j} + (p_n, c_n^L, c_n^R)x_{nj}, \qquad j = 1, 2, \ldots, m, \tag{4.34}$$

for the second type of the fuzzy parameters.

In the nonfuzzy output data regression described by equations (4.33) and (4.34), we seek to find the coefficients $A_i = (p_i,\ c_i)$ or $A_i = (P_i, c_i^L, c_i^R)$ that minimize the spread of the fuzzy output for all data sets. In mathematical form, this can be described as

Minimize

$$J_1 = \sum_{j=1}^{m}\sum_{i=1}^{n} \left| c_i x_{ij} \right| \tag{4.35}$$

such that the fuzzy regression model could contain all observed data in the estimated fuzzy numbers resulting from the model. This can be expressed mathematically as

$$y_j \geq \sum_{i=1}^{n} p_i x_{ij} - (1-\lambda)\sum_{i=1}^{n} c_i x_{ij} \tag{4.36}$$

$$y_j \leq \sum_{i=1}^{n} p_i x_{ij} + (1-\lambda)\sum_{i=1}^{n} c_i x_{ij} \tag{4.37}$$

Note that the first term on the right side of equations (4.36) and (4.37) represents the estimated middle of the fuzzy coefficients, whereas the second term represents the estimated spread of these coefficients and λ is the level of fuzziness and is specified by the user.

For the fuzzy coefficients of the *LR* type, the cost function to be minimized is

Minimize

$$J_1 = \sum_{j=1}^{m}\sum_{i=1}^{n} \left| \left(2m_j - 2p_j x_{ij} + c_i^L x_{ij} - c_i^R x_{ij}\right) \right| \tag{4.38}$$

subject to satisfying the following two constraints on each data point

$$y_j \geq \sum_{i=1}^{n} p_i x_{ij} - (1-\lambda)\sum_{i=1}^{n} c_i^L x_{ji}, \qquad j = 1, \ldots, \ldots, m \tag{4.39}$$

$$y_j \leq \sum_{i=1}^{n} p_i x_{ij} + (1-\lambda)\sum_{i=1}^{n} c_i^L x_{ji}, \qquad j = 1, \ldots, \ldots, m \tag{4.40}$$

The problems formulated in equations (4.35), (4.36), and (4.37) and formulated in equations (4.38), (4.39), and (4.40) are linear optimization problems, which can be solved by the well-known linear programming–based simplex method. However, if the sum of the absolute value deviations in equations (4.35) and (4.38) is to be minimized, subject to satisfying the inequality constraints given by equations (4.36) and (4.37) and equations (4.39) and (4.40), then the problem turns out to be one of the least absolute value linear optimization problems and can be solved by using the software package RLAV available in the IMSL/STAT library.

4.4.2 Fuzzy Output Systems

If the output is fuzzy, in this case it may be represented by a fuzzy number in the form $Y_j = (m_j, \alpha_j)$ in the case of a triangular fuzzy number (TFN) or $Y_j = (m_j, \alpha_j^L, \alpha_j^R)$, $j = 1, 2, \ldots, m$ in the case of a trapezoidal membership function. For the TFN membership function, equation (4.30) can be written as

$$Y_j = (m_j, \alpha_j) = (p_1, c_1)x_{1j} + (p_2, c_2)x_{2j} + (p_n, c_n)x_{nj} \qquad j = 1, 2, \ldots, m \tag{4.41}$$

which can be rewritten as

$$(m_j, \alpha_j) = (p_1x_{1j} + p_2x_{2j} + \cdots + p_nx_{nj}, c_1x_{1j} + c_2x_{2j} + c_nx_{nj}), \quad j = 1, 2, \ldots, m \tag{4.42}$$

$$(m_j, \alpha_j) = \left(\sum_{i=1}^{n} p_i x_{ij}, \sum_{i=1}^{n} c_i x_{ij} \right) \tag{4.43}$$

Equation (4.43) is valid when

$$m_j = \sum_{i=1}^{n} p_i x_{ij} \qquad j = 1, 2, \ldots, m \tag{4.44}$$

$$\alpha_j = \sum_{i=1}^{n} c_i x_{ij} \qquad j = 1, 2, \ldots, m \tag{4.45}$$

The problem now turns out to be as follows: Given the fuzzy output $Y_j = (m_j, \alpha_j)$, the task is to find the fuzzy parameters (p_i, c_i), $i = 1, 2, \ldots, n$ that minimize the cost function given by

$$J_1(p_i, c_i) = \sum_{j=1}^{m} \left| \left(m_j - \sum_{i=1}^{n} p_i x_{ij} + \alpha_j - \sum_{i=1}^{n} c_i x_{ij} \right) \right| \tag{4.46}$$

subject to satisfying the following constraints on each measurement point:

$$m_j - (1-\lambda)\alpha_j \geq \sum_{i=1}^{n} p_i x_{ij} - \sum_{i=1}^{n} c_i x_{ij} \qquad j = 1, 2, \ldots, n \tag{4.47}$$

$$m_j + (1-\lambda)\alpha_j \leq \sum_{i=1}^{n} p_i x_{ij} + \sum_{i=1}^{n} c_i x_{ij} \qquad j = 1, 2, \ldots, n \tag{4.48}$$

If the fuzzy output is of the LR type, then equation (4.42) can be rewritten as

$$(m_j, \alpha_j^L, \beta_j^R) = \left(\sum_{i=1}^{n} p_i x_{ij}, \sum_{i=1}^{n} c_i^L x_{ij}, \sum_{i=1}^{n} c_i^R x_{ij} \right) \tag{4.49}$$

Equation (4.49) can be separated into the following equations:

$$m_j = \sum_{i=1}^{n} p_i x_{ij} \qquad j = 1, \ldots, \ldots, \ldots, m \tag{4.50}$$

$$\alpha_j^L = \sum_{i=1}^{n} c_i^L x_{ij} \qquad j = 1, \ldots, \ldots, \ldots, m \tag{4.51}$$

$$\beta_j^R = \sum_{i=1}^{n} c_i^R x_{ij} \qquad j = 1, \ldots, \ldots, \ldots, m \tag{4.52}$$

In this case, the objective function to be minimized is given as

$$J_1 = \frac{1}{4} \sum_{i=1}^{n} \left| \left[4m_j - 4\sum_{i=1}^{n} p_i x_{xj} - \alpha_j^L + \sum_{i=1}^{n} c_i^L x_{xj} - \beta_j^R - \sum_{i=1}^{n} c_i^R x_{ij} \right] \right| \tag{4.53}$$

subject to satisfying the following constraints:

$$m_j - (1-\lambda)c_j^L \geq \sum_{i=1}^{n} p_i x_{ij} - \sum_{i=1}^{n} c_i^L x_{ij}, \qquad j = 1, 2, \ldots, n \tag{4.54}$$

$$m_j + (1-\lambda)c_j^R \leq \sum_{i=1}^{n} p_i x_{ij} + \sum_{i=1}^{n} c_j^R x_{ij}, \qquad j = 1, 2, \ldots, n \tag{4.55}$$

Again, the problems formulated in equations (4.46), (4.47), and (4.48) and those formulated in equations (4.53), (4.54), and (4.55) for LR type are all linear optimization problems subjected to a set of linear constraints. These problems can be solved using the standard linear programming–based simplex method. However, if the objective functions are the minimization of the sum of the absolute value of the deviation, then the least absolute value optimization technique based on linear programming is used to solve the problems formulated here.

Example 4.1

Consider the data set shown in Table 4.3. The task here is to find a fuzzy model in the form of $y_j = A_0 + A_1 x_{1j} + A_2 x_{2j}$; $j = 1, \ldots, 5$ that fits this set of data.

The output is nonfuzzy data. The objective function to be minimized is

$$J = c_0 + c_1 \sum_{j=1}^{m} x_{ij} + c_2 \sum_{j=1}^{m} x_{ij}$$
$$= c_0 + 3.68 c_1 + 2.05 c_2$$

subject to satisfying the following two constraints on each data point for $j = 1, 2, 5$:

$$y_j \geq (p_0 + p_1 x_{1j} + p_2 x_{2j}) - (1 - \lambda)(c_0 + c_1 x_{1j} + c_2 x_{2j})$$

$$y_j \leq (p_0 + p_1 x_{1j} + p_2 x_{2j}) + (1 - \lambda)(c_0 + c_1 x_{1j} + c_2 x_{2j})$$

The solution to the preceding linear programming problem using the simplex method is

$$A_0^* = (0.4391, 0.204), A_1^* = (0.0, 0.0), A_2^* = (6.963, 0.0)$$

With the cost function of $J = 0.204$, while the residual of each data point can be calculated as

$$Y_j^* = (0.4391,\ 0.204) + (6.963,\ 0.0) x_{2j}$$

the middle is

$$m_j^* = 0.4391 + 6.963 x_{2j} \qquad j = 1, \ldots, \ldots, 5$$

which gives a residual vector of

$$r^* = (-0.10208,\ -0.00986,\ 0.10199,\ -0.10200,\ 0.0683)^T$$

The preceding results are obtained when $\lambda = 0.5$ and the degree of fuzziness $= 0.5$.

Note that A_0^* is a fuzzy parameter because it has a spread of $c_0 = 0.204$, but the coefficients A_1^* and A_2^* are not fuzzy parameters.

Table 4.3 Five-Data Sample for Example 4.1

Y	x_1	x_2
3.54	0.84	0.46
4.05	0.65	0.52
4.51	0.76	0.57
2.63	0.70	0.30
1.90	0.73	0.20

Example 4.2

In a fuzzy regression, the output y is a TFN with c_j representing the error. The fuzzy output and the corresponding crisp input are given as:

(y_i, c_i)	x_i
(2.1, 0.2)	0.52
(4.6, 0.35)	1.36

where $\lambda = 0.4$ determines the fuzzy coefficients for a simple model $Y_j = A_0 + A_1 x_j$.

Because the output is a fuzzy number of TFN memberships, then the cost function to be minimized is

$$J = \sum_{j=1}^{2} (m_j - (p_0 + p_1 x_{1j}) + \alpha_j - (c_0 + c_1 x_{1j}))$$
$$= 7.25 - (p_0 + p_1 x_{1j})$$

Because the first term is a constant one, the cost function to be minimized is

$$J = -(p_0 + p_1 x_{1j} + c_0 + c_1 x_{1j})$$

subject to satisfying the following constraints:

$$m_j - 0.6\alpha_j \geq (p_0 + p_1 x_{1j}) - (c_0 + c_1 x_{1j}), \qquad j = 1, 2$$

$$m_j + 0.6\alpha_j \geq (p_0 + p_1 x_{1j}) + (c_0 + c_1 x_{1j}), \qquad j = 1, 2$$

Substituting the preceding data given we obtain

$$0.9 \geq p_0 + 0.52 p_1 - c_0 - 0.52 c_1$$

$$4.39 \geq p_0 + 1.36 p_1 - c_0 - 1.36 c_1$$

$$2.22 \leq p_0 + 0.52 p_1 + c_0 + 0.52 c_1$$

$$4.81 \leq p_0 + 1.36 p_1 + c_0 + 1.36 c_1$$

Using the linear programming–based simplex method approach, we obtain the following solution:

$$A_0^* = (2.855,\ 1.955),\ A_1^* = (0.0, 0.0)$$

where $J = 2.44$ and the residual vector of the inequality constraint is

$$r^* = (0.0,\ -3.49,\ -2.59,\ 0.00)^T$$

This indicates that the obtained solution is valid.

Example 4.3

This example is for the *LR* type; the solution obtained was based on minimum least absolute deviation. The data for the TFN fuzzy output are listed in Table 4.4.

The model required to fit these data points is in the form

$$Y_j = A_0 + A_1 x_j + A_1 x_j^2, \qquad j = 1, \ldots, \ldots, 16$$

The cost function to be minimized in this case is given as

$$J = 0.25\sum_{j=1}^{16}\left(4m_j - \alpha_j^L + \beta_j^R\right) - 0.25[(4p_0 + 96p_1 + 148.4p_2 - c_0^L - 24c_1^L - 96.6c_2^L + c_0^R + 24c_1^R + 96.6c_2^R)]$$

As mentioned earlier, if the first term of *J* is constant, then the cost function to be minimized is

$$J_1 = -0.25\left[(4p_0 + 96p_1 + 148.4p_2 - c_0^L - 24c_1^L - 96.6c_2^L + c_0^R + 24c_1^R + 96.6c_2^R)\right]$$

subject to satisfying the inequality constraints given as

$$m_j - (1-\lambda)c_j^L \geq \left(p_0 + p_1 x_{1j} + p_2 x_{2j}^2\right) - \left(c_0^L + c_1^L x_{1j} + c_2^L x_{2j}^2\right), \quad j = 1, \ldots, \ldots, 16$$

$$m_j + (1-\lambda)c_j^L \leq \left(p_0 + p_1 x_{1j} + p_2 x_{2j}^2\right) + \left(c_0^L + c_1^L x_{1j} + c_2^L x_{2j}^2\right), \quad j = 1, \ldots, \ldots, 16$$

Note that, the number of parameters to be estimated is 9 and the number of inequality constraints is 32. The solution to the fuzzy parameters for the proposed model has been found to be

$$A_0 = (12.75,\ 2.75,\ 0.0),\ \underline{A}_1 = (42.1, 0, 0),\ \text{and}$$
$$\underline{A}_2 = (140.794, 144.3, 0.0)$$

Table 4.4 Data for Example 4.3

No.	x_j	$Y_j = (m_j, \alpha_j^L, \beta_j^R)$	No.	x_j	$Y_j = (m_j, \alpha_j^L, \beta_j^R)$
1	0.0	(11.5, 3, 2.5)	9	1.6	(84., 15., 16.)
2	0.2	(24.8, 4.5, 4.)	10	1.8	(82., 15., 16.)
3	0.4	(40. 6., 7.)	11	2.0	(103.7, 16., 17.)
4	0.6	(45.2, 7., 7.)	12	2.2	(102.6, 16., 17.)
5	0.8	(49.1, 9. 9.)	13	2.4	(103.1, 16., 17.)
6	1.0	(70. 11. 12.)	14	2.6	(111., 17., 19.)
7	1.2	(70.9, 12. 12.)	15	2.8	(109., 17. 19.)
8	1.4	(80.1, 14. 15.)	16	3.0	(121.7, 18. 21.)

with an alarm from the linear program that this solution is not the unique solution. The model equation in this case can be written as

$$\underline{Y} = (12.75,\ 2.75,\ 0.0) + (42.1,\ 0,\ 0)x + (140.794,\ 144.3,\ 0.0)x^2$$

This model satisfies all the constraints on the fuzzy optimization problem formulated previously. In the next section, we offer an example for the electrical load estimation.

Example 4.4

The fuzzy linear parameter estimation algorithm, proposed in the preceding sections, is implemented for determination of the required distribution system under uncertain conditions [18]. The uncertainty appears at input, at output, and in the nature of the system itself. Measured data are given in Table 4.5, where E_r is the yearly energy consumption, P_i is the installed capacity of electrical equipment at customers' sites, and P_r is the yearly peak load.

The task is to build a fuzzy linear model that relates the yearly energy consumption E_r and P_r in the form of

$$P_r = \underline{A}_0 + \underline{A}_r E_r$$

or

$$P_r = (p_0, c_0) + (p_1, c_1)E_r$$

Table 4.5 Power Measured at a Substation for Example 4.4

#	E_r(MWh)	P_i(kW)	P_r(kW)
1	21.79	125	30.
2	60.20	247	27.9
3	60.72	436	40.5
4	65.01	406	39.6
5	70.00	265	42.0
6	70.55	251	29.2
7	72.30	520	42.0
8	79.05	540	42.0
9	80.39	310	30.9
10	114.0	443	57.0
11	114.45	573	57.5
12	125.00	438	37.2
13	148.00	578	55.5
14	162.10	610	59.6

The cost function to be minimized is

$$J_1 = c_0 + \sum_{j=1}^{14} c_1 E_{ir}$$

subject to satisfying the following two constraints:

$$P_{ri} \geq (p_0 + p_1 E_{ir}) - (1. - \lambda)(c_0 + c_1 E_{ir}), \qquad i = 1, 2, \ldots, \ldots, 14$$

$$P_{ri} \leq (p_0 + p_1 E_{ir}) + (1. - \lambda)(c_0 + c_1 E_{ir}), \qquad I = 1, 2, \ldots, \ldots, 14$$

The solution to this linear optimization problem when $\lambda = 0.5$ is

$$P_r = (29.76, 21.85) + (0.14692, 0.0)E_r$$

with $J = 21.85$. Note that $\underline{A}_0$ is a fuzzy number because it has a spread of 21.85, but $\underline{A}_1$ is a crisp number. By using this model, we notice that P_r is fuzzy data having a constant spread of 21.85 along the whole measurement. In other words, the yearly peak load P_r is a fuzzy load having a TFN membership with a middle given in the table and a spread of 21.85 kW.

If λ is chosen to be zero, then the following solution is obtained:

$$P_r = [29.756, 10.925] + [0.14692, 0.0]E_r$$

with $J = 10.925$; that is, the fitted middle model does not change at both values of λ, but as the degree of fuzziness decreases, the spread decreases.

Another test is conducted such that when we model P_r by a second-order model with E_r, it has been shown that the first-order model mentioned previously is adequate to model such a load because the fuzzy coefficient of the second-order term equals zero.

Example 4.5

In Example 4.4, we stated that it is required to model P_r as a function of P_i in a first-order model as

$$P_r = f(P_i) \text{ or}$$

$$P_r = \underline{A}_0 + \underline{A}_1 P_i$$

The cost function to be minimized in this case, according to the data available in Table 4.5, is

$$J_1 = c_0 + 5742\, c_i$$

subject to satisfying the following two constraints on the measurement set:

$$P_{rj} \geq (p_0 + p_1 P_{ij}) - (1. - \lambda)(c_0 + c_1 P_{ij}), \qquad j = 1, 2, \ldots, \ldots, 14$$

$$P_{rj} \leq (p_0 + p_1 P_{ij}) + (1. - \lambda)(c_0 + c_1 P_{ij}), \qquad j = 1, 2, \ldots, \ldots, 14$$

The results obtained in this test for $\lambda = 0.5$ are

$$P_r = (25.78, 19.6813) + (0.0483, 0.0) P_i$$

with $J = 19.6813$. It has been found that such a model is adequate for these data, and a higher-order model gives zero fuzzy coefficients.

If the yearly peak load P_r is presented as a function of E_r and P_i as

$$P_r = \underline{A}_0 + \underline{A}_1 E_r + \underline{A}_2 P_i$$

then the cost function to be minimized in this case is

$$J = c_0 + 1243.56 c_1 + 5742 c_2$$

subject to satisfying

$$(P_r)_j \geq p_0 + p_1 (E_r)_j + p_2 (P_i)_j - (1 - \lambda)(c_0 + c_1 (E_r)_j + c_2 (P_i)_j),$$
$$j = 1, \ldots, \ldots, 14$$

$$(P_r)_j \leq p_0 + p_1 (E_r)_j + p_2 (P_i)_j + (1. - \lambda)(c_0 + c_1 (E_r)_j + c_2 (P_i)_j),$$
$$j = 1, \ldots, \ldots, 14$$

The solution to the preceding optimization problem at $\lambda = 0.5$ is

$$P_r = (25.51, 15.59) + (0.0027, 0.00) E_r + (0.0483, 0.0) P_i$$

with $J = 19.59$. Note that $\underline{A}_0$ is a fuzzy number with a spread of 19.59 and that P_r is a fuzzy number with a spread $= 19.59$.

Example 4.6

The yearly peak load in the preceding example is given as an *LR* type, as shown in Table 4.6 [18]. The task is to model this load as

$$P_s = \underline{A}_0 + A_r E_r$$

where $\underline{A}_0 = \left(p_0, c_0^L, c_0^R\right)$ and $\underline{A}_1 = \left(p_1, c_1^L, c_1^R\right)$.

Using the cost function defined in equation (4.53) and the constraints defined in equations (4.54) and (4.55), for this linear model, we obtain the following results:

$$P_s = (0.0, 0.0, 0.0) + (2.134, 1.974, 0.0)E_r$$

Now, the spread of P_r at a given measurement *j,* is

$$\left(c_s^L, c_s^R\right) = (1.974E_r,\ 0.0)_j \qquad j = 1, \ldots, \ldots, 14$$

while the middle is

$$(m_s)_j = (2.134E_r)_j \qquad j = 1, \ldots, \ldots, 14$$

Table 4.6 Samples of Measurements in SS for Example 4.6

#	E_r(MWh)	P_i(kW)	P_r(kW)
1	21.79	125	(30, 25, 33)
2	60.20	247	(27.9, 24, 30.5)
3	60.72	436	(40.5, 34.8, 44.9)
4	65.01	406	(39.6, 36.1, 43)
5	70.00	265	(42.0, 38, 45.7)
6	70.55	251	(29.2, 26, 33)
7	72.30	520	(42.0, 37.6, 45)
8	79.05	540	(42.0, 38.5, 46)
9	80.39	310	(30.9, 27, 34.5)
10	114.0	443	(57.0, 52.5, 60.9)
11	114.45	573	(57.5, 52.8, 61.4)
12	125.00	438	(37.2, 34.4, 40.8)
13	148.00	578	(55.5, 52, 59.8)
14	162.10	610	(59.6, 55.3, 64.7)

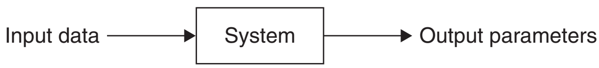
4.5 Fuzzy Short-Term Load Modeling

Most of the work on offline short-term load models available today assumes that the parameters of the model are constant crisp values [14–19]. This assumption is, to some extent, true, as long as there are no big changes in weather parameters from day to day. The load power is characterized by both uncertainty and ambiguity.

In this section, the load models used in Chapter 3 are reformulated to account for fuzziness of the load characteristics. In the first subsection the input is assumed to be crisp, whereas the load model parameters are expressed as fuzzy numbers having a certain middle and spreads. Three models are used in this section—namely, fuzzy load models A, B, and C. Fuzzy load model A is a multiple linear regression model. This model takes into account the weather parameters. Fuzzy load model B is a harmonic model and does not account for the weather parameters. Fuzzy load model C is a hybrid model that combines models A and B and takes into account the weather parameters.

In this section we assume that the input data are fuzzy numbers having certain middles and spreads. The parameters of the load model are fuzzy. The fuzzy numbers used for the fuzzy variables in this section are assumed to have a symmetrical triangular membership function.

The following system is considered:



If the input data are crisp (nonfuzzy) and the system parameters A_i $(i, 1, \ldots, n)$ are crisp (nonfuzzy), then the output is also crisp (nonfuzzy) with an error deviation between the actual and the estimated or predicted values. If the input data are crisp (nonfuzzy) and the system parameters are fuzzy and follow a membership function (e.g., a triangular membership function), then the output is fuzzy and follows the same membership as in the system parameters. If the input data are fuzzy and the system parameters are fuzzy, then the output is fuzzy. The output will have some resemblance of shape to the membership function used.

The membership functions used in this section are triangular membership functions with fuzzy numbers having a certain middle and equal left and right spreads. The objective of the fuzzy parameters estimation is to minimize the spreads of the fuzzy parameters. If spreads of zero are attained, then the output is crisp with an error deviation from the actual value. If the spreads are minimized, then the output will follow the shape of a triangular membership function and the output value will be in a range between upper and lower values.

4.5.1 Multiple Fuzzy Linear Regression Model: Crisp Data

$$(Y_j(t) = m_j(t), \qquad j = 1, \ldots, m;\ t = 1, 2, \ldots, \ldots, 24)$$

The input data of the load model are assumed to be crisp values, whereas the load parameters are fuzzy. The load, in this model, can be expressed mathematically as

$$Y_j(t) = \underline{A}_0 + \sum_{i=1}^{n} \underline{A}_i x_{ij}(t), \qquad j = 1, \ldots, m \tag{4.56}$$

where

$Y_j(t)$ is the value of the load power at time t;
$\underline{A}_0$ is the fuzzy base load having a triangular membership with a middle p_0 and spread c_0, as shown in Figure 4.6(a);
$\underline{A}_i$ are the fuzzy coefficients having a triangular membership with a middle p_i and spread c_i, as shown in Figure 4.6(b).

Equation (4.56) can be rewritten as

$$Y_j(t) = (p_{yj}(t),\, c_{yj}(t)) = m_j(t) = (p_0, c_0) + \sum_{i=1}^{n} (p_i, c_i)\, x_{ij}(t) \tag{4.57}$$

As shown earlier in this chapter, for the output data described by equation (4.57), the coefficients $\underline{A}_0$ (p_0, c_0) and $\underline{A}_i$ (p_i, c_i) are to be found such that the spread of the fuzzy output is minimized for all data sets. In mathematical form, this can be described as

Minimize

$$J = \left| \sum_t \left\{ c_0 + \sum_{j=1}^{m} \sum_{i=1}^{n} c_i x_{ij}(t) \right\} \right| \tag{4.58}$$

where $t \in [0,\ t_F]$, t_F is the number of days for which data are taken at the hour in question. The fuzzy regression model in equation (4.58) contains all observed data in the estimated fuzzy numbers resulting from the model. This can be expressed mathematically as

$$y_j(t) \geq \left[p_0 + \sum_{i=1}^{n} p_i x_{ij}(t) \right] - (1-\lambda) \left[c_0 + \sum_{i=1}^{n} c_i x_{ij}(t) \right]; \quad j = 1, \ldots, m \tag{4.59}$$

and

$$y_j(t) \leq \left[p_0 + \sum_{i=1}^{n} p_i x_{ij}(t) \right] + (1-\lambda) \left[c_0 + \sum_{i=1}^{n} c_i x_{ij}(t) \right]; \quad j = 1, \ldots, m \tag{4.60}$$

Note that the first term on the right side of equations (4.59) and (4.60) represents the estimated middle of the fuzzy coefficients, and the second term represents the estimated spread of these coefficients. λ is the level of fuzziness and is specified by the user. As λ increases, the fuzziness of the output increases. In the preceding equations, m is the number of observations, and n is the number of fuzzy parameters used in the model.

In the following subsections, two multiple fuzzy linear regression models are developed. The first model can be used to predict the load during the winter season, whereas the second model can be used to predict the load during the summer season. The only difference between the two models is that the winter model considers the

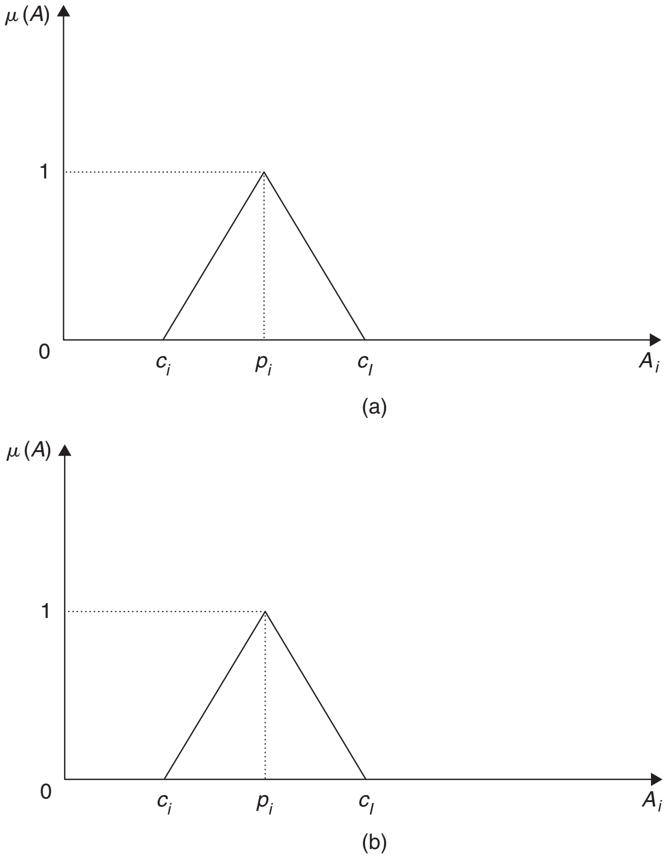


Figure 4.6 (a) Membership function of A_0; (b) membership function of A_I.

wind-cooling factor as an explanatory variable, and the summer model considers the humidity factor as an explanatory variable.

4.5.1.1 Fuzzy Load Model A: Winter Model

The fuzzy winter model, in Chapter 3, equation (3.12), can be rewritten in fuzzy form as

$$\begin{aligned} Y_j(t) = {} & \underline{A}_0 + \underline{A}_1 T_j(t) + \underline{A}_2 T_j^2(t) + \underline{A}_3 T_j^3(t) + \underline{A}_4 T_j(t-1) + \underline{A}_5 T_j(t-2) \\ & + \underline{A}_6 T_j(t-3) + \underline{A}_7 W_j(t) + \underline{A}_8 W_j(t-1) + \underline{A}_9 W_j(t-2);\ j = 1, \ldots, m \end{aligned} \tag{4.61}$$

where $Y_j(t)$ is the load power j; $j = 1, \ldots, m$ at time t; $t = 1, 2, \ldots, 24$ and is assumed to be given as nonfuzzy data. $T_j(t)$ is the jth temperature deviation from nominal at time t and is given by equation (3.13). $W_j(t)$ is the wind-cooling factor at time t and is given by equation (3.15), and $\underline{A}_0, \underline{A}_1, \ldots, \underline{A}_9$ are load model fuzzy coefficients having middles $p_0, p_1, \ldots, p_9$ and spreads $c_0, c_1, \ldots, c_9$.

Equation (4.61) can be rewritten as

$$\begin{aligned} Y_j(t) = {} & (p_0, c_0) + (p_1, c_1)T_j(t) + (p_2, c_2)\, T_j^2(t) + (p_3, c_3)\, T_j^3(t) \\ & + (p_4, c_4)T_j(t-1) + (p_5, c_5)T_j(t-2) + (p_6, c_6)T_j(t-3) \\ & + (p_7, c_7)W_j(t) + (p_8, c_8)W_j(t-1) + (p_9, c_9)W_j(t-2); \\ & j = 1, \ldots, m \end{aligned} \tag{4.62}$$

In fuzzy linear regression, the spreads of the fuzzy coefficients are to be minimized. This results in an objective function that can be expressed mathematically as

$$\begin{aligned} J = \sum_t \Bigg\{ c_0 + \sum_{j=1}^{m} \Big[& c_1 T_j(t) + c_2 T_j^2(t) + c_3 T_j^3(t) + c_4 T_j(t-1). \\ & + c_5 T_j(t-2) c_6 T_j(t-3) + c_7 W_j(t) + c_8 W_j(t)(t-1) + c_9 W_j(t-2) \Big] \Bigg\} \end{aligned} \tag{4.63}$$

where $t \in [0, t_F]$, t_F is the number of days for which data are taken at the hour in question. This is subject to satisfying the two inequality constraints on each load power given as

$$\begin{aligned} y_j(t) \geq {} & p_0 + p_1 T_j(t) + p_2 T_j^2(t) + p_3 T_j^3(t) + p_4 T_j(t-1) + p_5 T_j(t-2) \\ & + p_6 T_j(t-3) + p_7 W_j(t) + p_8 W_j(t-1) + p_9 W_j(t-2) - (1-\lambda)(c_0 + c_1 T_j(t) \\ & + c_2 T_j^2(t) + c_3 T_j^3(t) + c_4 T_j(t-1) + c_5 T_j(t-2) + c_6 T_j(t-3) \\ & + c_7 W_j(t) + c_8 W_j(t-1) + c_9 W_j(t-2)), \quad j = 1, 2, \ldots, m \end{aligned} \tag{4.64}$$

$$\begin{aligned} y_j(t) \leq {} & p_0 + p_1 T_j(t) + p_2 T_j^2(t) + p_3 T_j^3(t) + p_4 T_j(t-1) + p_5 T_j(t-2) \\ & + p_6 T_j(t-3) + p_7 W_j(t) + p_8 W_j(t-1) + p_9 W_j(t-2) \\ & + (1-l)(c_0 + c_1 T_j(t) + c_2 T_j^2(t) + c_3\, T_j^3(t) + c_4 T_j(t-1) \\ & + c_5 T_j(t-2) + c_6 T_j(t-3) + c_7 W_j(t) + c_8 W_j(t-1) \\ & + c_9 W_j(t-2)), \; j = 1, 2, \ldots, m \end{aligned} \tag{4.65}$$

The optimization problem formulated in equations (4.63) to (4.65) is linear and can be solved using linear programming based on the simplex method available in the IMSL/STAT library.

Having identified the middle and spread of each coefficient, we can then obtain the fuzzy load model for the winter season using equation (4.61) or equation (4.62).

4.5.1.2 Fuzzy Load Model A: Summer Model

The summer fuzzy model for short-term load forecasting can be written as

$$\begin{aligned} Y(t) = {} & \underline{A}_0 + \underline{A}_1 T(t) + \underline{A}_2 T^2(t) + \underline{A}_3 T^3(t) + \underline{A}_4 T(t-1) + \underline{A}_5 T(t-2) \\ & + \underline{A}_6 T(t-3) + \underline{A}_7 H(t) + \underline{A}_8 H(t-1) + \underline{A}_9 H(t-2) \end{aligned} \tag{4.66}$$

where

$\underline{Y}(t)$ is the summer load power at time t;
$T(t)$ is the temperature deviation at time t given by equation (3.13) in Chapter 3;
$\underline{A}_0, \underline{A}_1, \ldots, \underline{A}_9$ are the fuzzy load coefficients having a certain middle $p_0, p_1, \ldots, p_9$ and certain spread $c_0, c_1, \ldots, c_9$ at time t;
$H(t)$ is the temperature humidity factor given by equation (3.17) in Chapter 3.

The summer load model stated in equation (4.66) takes into account the temperature deviation and the temperature humidity factor for each hour and at three and two hours before.

Equation (4.66) can be rewritten as

$$\begin{aligned} Y_j(t) = {} & (p_0, c_0) + (p_1, c_1)\, T_j(t) + (p_2, c_2)\, T_j^2(t) + (p_3, c_3)\, T_j^3(t) \\ & + (p_4, c_4)\, T_j(t-1) + (p_5, c_5)\, T_j(t-2) + (p_6, c_6)\, T_j(t-3) \\ & + (p_7, c_7)\, H_j(t) + (p_8, c_8)\, H_j(t-1) + (p_9, c_9)\, H_j(t-2) \end{aligned} \tag{4.67}$$

In fuzzy linear regression, the parameters $\underline{A}_i = (p_i, c_i)$, $i = 1, \ldots, \ldots, 9$ are to be found that minimize the spread of the fuzzy output for all data sets. This can be expressed mathematically as

Minimize

$$\begin{aligned} J = \Bigg| \sum_t \Bigg\{ c_0 + \sum_{j=1}^{m} \Big[& c_1 T_j(t) + c_2 T_j^2(t) + c_3 T_j^3(t) + c_4 T_j(t-1) \\ & + c_5 T_j(t-2) + c_6 T_j(t-3) + c_7 H_j(t) + c_8 H_j(t-1) + c_9 H_j(t-2) \Big] \Bigg\} \Bigg| \end{aligned} \tag{4.68}$$

where $t \in [0, t_F]$, t_F is the number of days for which data are taken at the hour in question. This is subject to satisfying the following inequality constraints at j; $j = 1, \ldots, m$:

$$\begin{aligned} y_j(t) \geq {} & p_0 + p_1 T_j(t) + p_2 T_j^2(t) + p_3 T_j^3(t) + p_4 T_j(t-1) + p_5 T_j(t-2) \\ & + p_6 T_j(t-3) + p_7 H_j(t) + p_8 H_j(t-1) + p_9 H_j(t-2) \\ & - (1-\lambda)\big[c_0 + c_1 T_j(t) + c_2 T_j^2(t) + c_3 T_j^3(t) + c_4 T_j(t-1) + c_5 T_j(t-2). \\ & + c_6 T_j(t-3) + c_7 H_j(t) + c_8 H_j(t-1) + c_9 H_j(t-2) \big], \quad j = 1, 2, \ldots, m \end{aligned} \tag{4.69}$$

$$\begin{aligned} y_j(t) \leq\, & p_0 + p_1 T_j(t) + p_2 T_j^2(t) + p_3 T_j^3(t) + p_4 T_j(t-1) + p_5 T_j(t-2) \\ & + p_6 T_j(t-3) + p_7 H_j(t) + p_8 H_j(t-1) + p_9 H_j(t-2) \\ & - (1-\lambda)\big[c_0 + c_1 T_j(t) + c_2 T_j^2(t) + c_3 T_j^3(t) + c_4 T_j(t-1) + c_5 T_j(t-2). \\ & + c_6 T_j(t-3) + c_7 H_j(t) + c_8 H_j(t-1) \ + c_9 H_j(t-2)\big], \quad j = 1, 2, \ldots, m \end{aligned} \tag{4.70}$$

The problem formulated in equations (4.68) to (4.70) is linear and can be solved by the linear programming optimization package available in the IMSL/STAT library.

Having obtained the fuzzy parameters $\underline{A}_i = (p_i, c_i),\ i = 1, \ldots, 9,$ we can then predict the load for the next 24 hours using equation (4.67).

4.5.1.3 Fuzzy Load Model B

Fuzzy load model B is a harmonic decomposition model and does not account for weather conditions. It does not account for temperature deviation, wind-cooling factor, or humidity factor. Thus, this model can be used for both winter and summer simulations.

The fuzzy load at any time t, therefore, can be written as

$$Y(t) = \underline{A}_0 + \sum_{i=1}^{n} (\underline{A}_i \sin i\omega t + \underline{B}_i \cos i\omega t) \tag{4.71}$$

where

$\underline{Y}(t)$ is the load power at time t and it is assumed to have crisp values;
$\underline{A}_0$, $\underline{A}_i$, and $\underline{B}_i$ are fuzzy parameters having certain middles and spreads and are given as $\underline{A}_0 = (p_0, c_0)$, $\underline{A}_i = (p_i,\ c_i)$, and $\underline{B}_i = (a_i,\ b_i)$.

The model described in equation (4.71) can be rewritten as

$$Y(t) = (p_0,\ c_0) + \sum_{i=1}^{n} [(p_i, c_i) \sin i\omega t + (\alpha_i, b_i) \cos i\omega t] \tag{4.72}$$

Note that the middles and the spreads are constants and are estimated seven times weekly.

The objective is to find the fuzzy parameters that minimize the spread of the load power. Mathematically, this can be written as

Minimize

$$J = \left| \sum_t \left\{ c_0 + \sum_{j=1}^{m} \sum_{i=1}^{n} \left[c_i x_{ij}(t) + b_i y_{ij}(t) \right] \right\} \right| \tag{4.73}$$

where

$x_{ij}(t) = (\sin\ i\omega t)_j, \qquad j = 1, \ldots, m;\ i = 1, \ldots, n;$
$y_{ij}(t) = (\cos\ i\omega t)_j, \qquad j = 1, \ldots, m;\ i = 1, \ldots, n;$

m, n are the number of observations and harmonics chosen in the model, respectively; $t \in [0, t_F]$, t_F is the number of days for which data are taken at the hour in question.

This is subject to satisfying the inequality constraints given by

$$
\begin{aligned}
y_j(t) \geq & \left[p_0 + \sum_{i=1}^{n} (p_i \sin i\omega t + \alpha_i \cos i\omega t) \right]_j \\
& - (1-\lambda) \left[c_0 + \sum_{i=1}^{n} (c_i \sin i\omega t + b_i \cos i\omega t)_j \right]
\end{aligned}
\tag{4.74}
$$

$$
\begin{aligned}
y_j(t) \leq & \left[p_0 + \sum_{i=1}^{n} (p_i \sin i\omega t + \alpha_i \cos i\omega t) \right]_j \\
& + (1-\lambda) \left[c_0 + \sum_{i=1}^{n} (c_i \sin i\omega t + b_i \cos i\omega t)_j \right]
\end{aligned}
\tag{4.75}
$$

The optimization problem formulated in equations (4.73) to (4.75) is a linear optimization problem and can be solved using the simplex method of linear programming.

Having obtained the fuzzy load parameters, we can then predict the load for the next 24 hours using equation (4.72).

4.5.1.4 Fuzzy Load Model C

Fuzzy load model C is a fuzzy hybrid model that takes into account weather-dependent components. The base load in the model is a time-varying function and takes the form of Fourier's coefficients. This model can be considered as a combination of fuzzy load model A and fuzzy load model B. Here, the weather input is limited only to temperature deviation, and the model is used for both winter and summer load forecast simulations.

The fuzzy load model in this case can be written mathematically as

$$
\begin{aligned}
Y_j(t) = & \left\{ \underline{A}_0 + \sum_{i=1}^{n} [\underline{A}_\mathrm{i} \sin\, i\omega t + \underline{B}_\mathrm{i} \cos\, i\omega t] \right\}_j \\
& + \left\{ \underline{C}_0 T_j(t) + \underline{C}_1 T_j(t-1) + \underline{C}_2 T_j(t-2) + \underline{C}_3 T_j(t-3) \right\}
\end{aligned}
\tag{4.76}
$$

where

$\underline{A}_0$, $\underline{A}_i$, $\underline{B}_i$ and are the weather-independent fuzzy parameters having certain middles and certain spreads;

$\underline{C}_0$, $\underline{C}_1$, $\underline{C}_2$, and $\underline{C}_3$ are the temperature-dependent fuzzy parameters.

The terms in the first brace in equation (4.76) can be considered as the base load, which depends only on time, whereas the terms in the second brace are the temperature-dependent load terms.

Equation (4.76) can be rewritten as

$$\begin{aligned} Y(t) = (p_0, c_0) + \sum_{i=1}^{n} [(p_i, \alpha_i) x_i(t) + (b_i, \beta_i)\, y_i(t)] + [(\gamma_o, s_0) T_j(t) \\ + (\gamma_1, s_1)\, T_j(t-1) + (\gamma_2, s_2)\, T_j(t-2) + (\gamma_3, s_3)\, T_j(t-3)] \end{aligned} \tag{4.77}$$

In equation (4.77), the first letter in the parameter's brackets indicates the middle of that parameter, and the second letter indicates the spread of this parameter.

In fuzzy regression, the fuzzy model parameters are to be found to minimize the spread of the output. In mathematical form, this can be expressed as

$$\begin{aligned} J = \Bigg| \sum_t \Bigg\{ c_0 + \sum_{j=1}^{m} \sum_{i=1}^{n} [\alpha_i x_{ij}(t) + \beta_i y_{ij}(t)] + \sum_{j=1}^{m} [s_0 T_j(t) + s_1 T_j(t-1) \\ + s_2 T_j(t-2) + s_3 T_j(t-3)] \Bigg\} \Bigg| \end{aligned} \tag{4.78}$$

where $t \in [0, t_F]$, t_F is the number of days for which data are taken at the hour in question.

This is subject to satisfying the following two constraints on the output so that the fuzzy regression model could contain all the observed data j, $j = 1, \ldots, m$ in the estimated fuzzy numbers resulting from the model. This can be expressed mathematically as

$$\begin{aligned} y_j(t) \geq \Bigg[p_0 + \sum_{i=1}^{n} (p_i x_{ij}(t) + b_i y_{ij}(t) + \gamma_0 T_j(t) + \gamma_1 T_j(t-1) + \gamma_2 T_j(t-2) \\ + \gamma_3 T_j(t-3)) \Bigg] - (1-\lambda) \Bigg[c_0 + \sum_{i=1}^{n} (\alpha_i x_{ij}(t) + \beta_i y_{ij}(t)) + s_0 T_j(t) \\ + s_1 T_j(t-1) + s_2 T_j(t-2) + s_3 T_j(t-3) \Bigg], \qquad j = 1, \ldots, m \end{aligned} \tag{4.79}$$

$$\begin{aligned} y_j(t) \leq \Bigg[p_0 + \sum_{i=1}^{n} (p_i x_{ij}(t) + b_i y_{ij}(t) + \gamma_0 T_j(t) + \gamma_1 T_j(t-1) + \gamma_2 T_j(t-2) \\ + \gamma_3 T_j(t-3)) \Bigg] + (1-\lambda) \Bigg[c_0 + \sum_{i=1}^{n} (\alpha_i x_{ij}(t) + \beta_i y_{ij}(t)) + s_0 T_j(t) \\ + s_1 T_j(t-1) + s_2 T_j(t-2) + s_3 T_j(t-3) \Bigg], \qquad j = 1, \ldots, m \end{aligned} \tag{4.80}$$

The problem formulated in equations (4.78) to (4.80) is a linear optimization problem and can be solved using linear programming based on the simplex method explained earlier in this chapter. Having identified the fuzzy model parameters, we can predict the load for the next 24 hours using equation (4.77).

4.5.2 Multiple Fuzzy Linear Regression Model: Fuzzy Data

In Section 4.5, the load power data are assumed to be nonfuzzy, whereas the parameters of the load power are fuzzy. Different linear optimization problems were derived with different load models. In this section, the load data are assumed to be fuzzy power values having a certain middle and certain spread $\underline{Y}_j(t) = [m_j(t), \alpha_j(t)]$, where $m_j(t)$ is the middle of the load power at the time t in question during the observation j, and $\alpha_j(t)$ is the spread of the load power at time t and observation j. Using this formulation of fuzzy numbers means that a triangular membership function is assumed, as shown in Figures 4.6(a) and (b).

The fuzzy model for the load power can be expressed mathematically as

$$Y_j(t) = [m_j(t), \alpha_j(t)] = \underline{A}_0 + \sum_{i=1}^{n} \underline{A}_i x_{ij}(t), \qquad j = 1, \ldots, m \tag{4.81}$$

which can be rewritten as

$$[m_j(t), \alpha_j(t)] = (p_0, c_0) + \sum_{i=1}^{n} (p_i, c_i)\, x_{ij}(t), \qquad j = 1, \ldots, m \tag{4.82a}$$

Alternatively, it can be separated as

$$[m_j(t), \alpha_j(t)] = \left[\left\{p_0 + \sum_{i=1}^{n} p_i x_{ij}(t)\right\}, \left\{c_0 + \sum_{i=1}^{n} c_i\, x_{ij}(t)\right\}\right], \qquad j = 1, \ldots, m \tag{4.82b}$$

Equation (4.82b) is valid only when: Given two fuzzy numbers $M_1 = (m_1, \alpha_1, \beta_1)_{\mathrm{LR}}$ and $M_2 = (m_2, \alpha_2, \beta_2)_{\mathrm{LR}}$ in terms of *LR* functions [2] that follow triangular membership function, where:

m_1 and m_2 are the centers of the membership function;
α_1 and α_2 are left-side spreads;
β_1 and β_2 are right-side spreads.

Then

$$M_1(m_1, \alpha_1, \beta_1)_{\mathrm{LR}} + M_2(m_2, \alpha_2, \beta_2)_{\mathrm{LR}} = (m_s, \alpha_s, \beta_s)_{\mathrm{LR}}$$

where

$$m_s = m_1 + m_2$$

$$\alpha_s = \alpha_1 + \alpha_2$$

$$\beta_s = \beta_1 + \beta_2$$

The center of the sum is equal to the sum of the centers, and each spread of the sum is the sum of its respective spread.

$$m_j(t) = p_0 + \sum_{i=1}^{n} p_i\, x_{ij}(t), \qquad j = 1, \ldots, \ldots, m \tag{4.83}$$

$$\alpha_j(t) = c_0 + \sum_{i=1}^{n} c_i\, x_{ij}(t), \qquad j = 1, \ldots, \ldots, m \tag{4.84}$$

The problem turns out to be: Given the fuzzy load power at time t, $\underline{Y}_j(t) = [m_j(t),\ \alpha_j(t)]$, the task is to find the fuzzy parameters $\underline{A}_0$ and $\underline{A}_i$ that minimize the cost function given by

$$J = \left| \sum_t \left\{ \sum_{j=1}^{m} \left\{ m_j(t) - p_0 - \sum_{i=1}^{n} p_i\, x_{ij}(t) + \alpha_j(t) - c_0 - \sum_{i=1}^{n} c_i x_{ij}(t) \right\} \right\} \right| \tag{4.85}$$

where $t \in [0,\ t_F]$, and t_F is the number of days for which data are taken at the hour in question.

This is subject to satisfying the following constraints on each measurement point

$$m_j(t) - (1-\lambda)\,\alpha_j(t) \geq \left(p_0 + \sum_{i=1}^{n} x_{ij}(t) \right) - \left(c_0 + \sum_{i=1}^{n} c_i\, x_{ij}(t) \right), \quad j = 1, \ldots, m \tag{4.86}$$

$$m_j(t) + (1-\lambda)\,\alpha_j(t) \leq \left(p_0 + \sum_{i=1}^{n} x_{ij}(t) \right) - \left(c_0 + \sum_{i=1}^{n} c_i\, x_{ij}(t) \right), \quad j = 1, \ldots, m \tag{4.87}$$

The problem formulated in equations (4.85) to (4.87) is a linear optimization problem. This problem can be solved using linear programming. In the following subsections, we discuss two multiple linear regression models: one for winter and one for summer.

4.5.2.1 Model A: Fuzzy Winter Model

Two factors affect this fuzzy winter model. The first is temperature deviation. The more temperature deviation, the more load power is needed. The second factor is wind cooling. As the wind-cooling factor increases, the load power increases. The load power data in this model are assumed to be a fuzzy power, unlike the load model in equation (4.62), where the load power is assumed to be crisp (nonfuzzy). Equation (4.62) can be rewritten as

$$\begin{aligned} Y_j(t) = (m_j(t), \alpha_j(t)) = (p_0, c_0) &+ (p_1, c_1)T_j(t) + (p_2, c_2)T_j^2(t) + (p_3, c_3)T_j^3(t) \\ &+ (p_4, c_4)T_j(t-1) + (p_5, c_5)T_j(t-2) + (p_6, c_6)T_j(t-3) \\ &+ (p_7, c_7)W_j(t) + (p_8, c_8)W_j(t-1) + (p_9, c_9)W_j(t-2) \end{aligned} \tag{4.88}$$

Equation (4.88) can be rewritten as

$$\begin{aligned} m_j(t) &= p_0 + p_1 T_j(t) + p_2 T_j^2(t) + p_3 T_j^3(t) + p_4 T_j(t-1) + p_5 T_j(t-2) \\ &\quad + p_6 T_j(t-3) + p_7 W_j(t) + p_8 W_j(t-1) + p_9 W_j(t-2), \\ &\quad j = 1, \ldots, m \end{aligned} \tag{4.89}$$

$$\begin{aligned} \alpha_j(t) &= c_0 + c_1 T_j(t) + c_2 T_j^2(t) + c_3 T_j^3(t) + c_4 T_j(t-1) + c_5 T_j(t-2) \\ &\quad + c_6 T_j(t-3) + c_7 W_j(t) + c_8 W_j(t-1) + c_9 W_j(t-2), \\ &\quad j = 1, \ldots, m \end{aligned} \tag{4.90}$$

Given the fuzzy load power $(m_j(t), \alpha_j(t))$ at any time t, the task is to determine the middle and the spread of each parameter that minimizes the cost function

$$\begin{aligned} J = \Bigg| \sum_t \sum_{j=1}^{m} [m_j(t) &- \{p_0 + p_1 T_j(t) + p_2 T_j^2(t) + p_3 T_j^3(t) + p_4 T_j(t-1) \\ &+ p_5 T_j(t-2) + p_6 T_j(t-3) + p_7 W_j(t) + p_8 W_j(t-1) + p_9 W_j(t-2)\} \\ &+ \alpha_j(t) - \{c_0 + c_1 T_j(t) + c_2 T_j^2(t) + c_3 T_j^3(t) + c_4 T_j(t-1) + c_5 T_j(t-2) \\ &+ c_6 T_j(t-3) + c_7 W_j(t) + c_8 W_j(t-1) + c_9 W_j(t-2)\}] \Bigg| \end{aligned} \tag{4.91}$$

where $t \in [0, t_F]$, and t_F is the number of days for which data are taken at the hour in question.

This is subject to satisfying the following two constraints at each measurement point:

$$\begin{aligned} m_j(t) - (1-\lambda)\,\alpha_j(t) &\geq [(\text{RHS of equation 4.89}) - (\text{RHS of equation 4.90})], \\ &\quad j = 1, \ldots, m \end{aligned} \tag{4.92}$$

$$\begin{aligned} m_j(t) + (1-\lambda)\,\alpha_j(t) &\leq [(\text{RHS of equation 4.89}) + (\text{RHS of equation 4.90})], \\ &\quad j = 1, \ldots, m \end{aligned} \tag{4.93}$$

where RHS stands for right-hand side.

The problem formulated in equations (4.91) to (4.93) is one of linear optimization. This problem can be solved using standard linear programming.

Having identified the fuzzy parameters of the fuzzy winter model, we can predict the load in a winter day. The middle of the load can be predicted at any hour t using equation (4.89), and the spread can be predicted using equation (4.90).

4.5.2.2 Model A: Fuzzy Summer Model

The load in the fuzzy summer model is a function of the temperature deviation and humidity factor. The load power and the load model parameters are assumed to be fuzzy numbers. Mathematically, this can be expressed as

$$\begin{aligned} Y_j(t) = (m_j(t),\, \alpha_j(t)) = \underline{A}_0 + \underline{A}_1 T_j(t) + \underline{A}_2 T_j^2(t) + \underline{A}_3 T_j^3(t) + \underline{A}_4 T_j(t-1) \\ + \underline{A}_5 T_j(t-2) + \underline{A}_6 T_j(t-3) + \underline{A}_7 H_j(t) + \underline{A}_8 H_j(t-1) + \underline{A}_9 H_j(t-2), \\ j = 1, \ldots, m \end{aligned} \tag{4.94}$$

where

$\underline{Y}_j(t)$ is the fuzzy load power i; $i = 1, \ldots, m$, at time t. This power has a middle $m_j(t)$ and a spread $\alpha_j(t)$;
$\underline{A}_0, \underline{A}_1, \ldots, \underline{A}_9$ are the fuzzy load parameters at time t with certain middle $p_0, \ldots, p_9$ and certain spread $c_0, c_1, \ldots, c_9$;
$T_j(t)$ is the temperature deviation at time t, $j = 1, \ldots, m$;
$H_j(t)$ is the humidity factor given by equation (3.17).

Equation (4.94) can be rewritten as

$$\begin{aligned} Y(t) = (m_j(t),\, \alpha_j(t)) = (p_0,\, c_0) + (p_1, c_1)\, T_j(t) + (p_2,\, c_2)\, T^2(t) \\ + (p_3,\, c_3) T^3(t) + (p_4,\, c_4)\, T(t-1) + (p_5, c_5)\, T(t-2) \\ + (p_6,\, c_6)\, T(t-3) + (p_7,\, c_7)\, H(t) + (p_8,\, c_8) H(t-1) \\ + (p_9,\, c_9)\, H(t-2) \end{aligned} \tag{4.95}$$

provided that the memberships for the fuzzy numbers are triangular memberships. Equation (4.91) can be rewritten as two equations:

$$\begin{aligned} m_j(t) = p_0 + p_1 T_j(t) + p_2 T_j^2(t) + p_3 T_j^3(t) + p_4 T_j(t-1) + p_5 T_j(t-2) \\ + p_6 T_j(t-3) + p_7 H_j(t) + p_8 H_j(t-1) + p_9 H_j(t-2), \\ j = 1, \ldots, m \end{aligned} \tag{4.96}$$

$$\begin{aligned} \alpha_j(t) = c_0 + c_1 T_j(t) + c_2 {T_j}^2(t) + c_3 {T_j}^3(t) + c_4 T_j(t-1) + c_5 T_j(t-2) \\ + c_6 T_j(t-3) + c_7 H_j(t) + c_8 H_j(t-1) + c_9 H_j(t-2), \\ j = 1, \ldots, m \end{aligned} \tag{4.97}$$

In the fuzzy optimization linear problem, the model fuzzy parameters are to be found to minimize the spread of the fuzzy load power. Mathematically, this can be expressed as

Minimize

$$J = \left| \sum_t \left\{ \sum_{j=1}^{m} \left[(m_j(t) - \text{RHS of equation 4.92}) + (\alpha_j(t) - \text{RHS of equation 4.93}) \right] \right\} \right| \tag{4.98}$$

where $t \in [0,\, t_F]$, and t_F is the number of days for which data are taken at the hour in question.

This is subject to satisfying the following constraints:

$$m_j(t) - (1-\lambda)\,\alpha_j(t) \geq [(\text{RHS of equation 4.92}) - (\text{RHS of equation 4.93})],\quad j = 1, \ldots, m \tag{4.99}$$

$$m_j(t) + (1-\lambda)\,\alpha_j(t) \leq [(\text{RHS of equation 4.92}) + (\text{RHS of equation 4.93})],\quad j = 1, \ldots, m \tag{4.100}$$

The optimization problem formulated in equations (4.98) to (4.100) is one of linear optimization and can be solved using linear programming.

Having obtained the fuzzy load parameters, we then can use equation (4.91) to predict the fuzzy load power at any hour t in question.

4.5.3 Fuzzy Load Model B

Fuzzy load model B does not account for weather conditions in the load; it can be expressed as

$$Y_j(t) = (m_j(t),\, \alpha_j(t)) = \underline{A}_0 \sum_{i=1}^{n} [(\underline{A}_{\mathrm{i}} \sin i\omega t + \underline{B}_{\mathrm{i}} \cos i\omega t]_j, \qquad j = 1, \ldots, m \tag{4.101}$$

The only difference between equation (4.71) and (4.101) is the load power $Y_j(t)$ at time t. In (4.71) the load power is assumed to be a crisp value, whereas in (4.101) it is assumed to be a fuzzy value having a middle $m_j(t)$ and a spread $\alpha_j(t)$. Equation (4.101) can be rewritten as

$$(m_j(t),\, \alpha_j(t)) = (p_0,\, c_0) + \sum_{i=1}^{n} [(p_i,\, c_i) \sin i\omega t + (b_i,\, \beta_i) \cos i\omega t]_j \quad j = 1, \ldots, m \tag{4.102}$$

which can be split into

$$m_j(t) = p_0 + \sum_{i=1}^{n} [(p_i \sin i\omega t + b_i \cos i\omega t)]_j, \qquad j = 1, \ldots, m \tag{4.103}$$

$$\alpha_j(t) = c_0 + \sum_{i=1}^{n} [c_i \sin i\omega t + \beta_i \cos i\omega t]_j, \qquad j = 1, \ldots, m \tag{4.104}$$

The task is to find the fuzzy load parameters that minimize the spread of the fuzzy load power. This can be expressed mathematically as

$$J = \left| \sum_t \left\{ \sum_{j=1}^{m} [(m_j(t) - \text{RHS of equation } 4.103 + (\alpha_j(t) - \text{RHS of equation } 4.104)] \right\} \right| \tag{4.105}$$

where $t \in [0,\ t_F]$, and t_F is the number of days for which data are taken at the hour in question.

This is subject to satisfying the following two constraints as

$$m_j(t) - (1-\lambda)\,\alpha_j(t) \geq [\text{RHS of equation } 4.103 - \text{RHS of equation } 4.104]; \quad j = 1, \ldots, m \tag{4.106}$$

$$m_j(t) + (1-\lambda)\,\alpha_j(t) \leq [\text{RHS of equation } 4.103 + \text{RHS of equation } 4.104]; \quad j = 1, \ldots, m \tag{4.107}$$

The problem formulated in equations (4.105) to (4.107) is one of linear optimization that can be solved using linear programming. Having identified the middle and the spread of fuzzy parameters, we then can use the harmonic load model described in equation (4.101) to predict the load at any hour t. Note that the load power obtained in this case is independent of the weather conditions and depends only on the hour in question.

The next model, model C, combines fuzzy load model A and fuzzy load model B. This model takes weather conditions into account.

4.5.4 Fuzzy Load Model C

Fuzzy load model A derived earlier has the advantage of being weather responsive; the fuzzy coefficients of this model depend on the weather conditions. These conditions include temperature deviation and cooling factor.

Fuzzy load model B is weather insensitive. The fuzzy coefficients of this model depend only on the time in question.

In this section, the two models A and B are combined into one fuzzy model, C. The resulting fuzzy load model C is weather sensitive. This fuzzy model is suitable for all weekdays and can be used for both winter and summer load-forecast simulations. Its main disadvantage is the assumption that the relation between load and weather is constant throughout the day.

The fuzzy model for the load in this case can be expressed mathematically as

$$\begin{aligned} Y_j(t) = (m_j(t),\ \alpha_j(t)) &= \left\{ \underline{A}_0 + \sum_{i=1}^{n} (\underline{A}_i \sin i\omega t + \underline{B}_i \cos i\omega t) \right\}_j \\ &+ \left\{ \underline{C}_0 T_j(t) + \underline{C}_1 T_j(t-1) + \underline{C}_2 T_j(t-2) + \underline{C}_3 T_j(t-3) \right\}_j, \\ & j = 1, \ldots, m \end{aligned} \tag{4.108}$$

where

$m_j(t)$, $\alpha_j(t)$ is the middle and spread of load power j, $j = 1, \ldots, m$ at time t;
$\underline{A}_0$, $\underline{A}_i$, and $\underline{B}_i$ are the weather-independent fuzzy parameters with certain middles and spreads;
$\underline{C}_0$, $\underline{C}_1$, $\underline{C}_2$, and $\underline{C}_3$ are the temperature-dependent fuzzy parameters with certain middles and spreads.

The left-hand side (LHS) of equation (4.108) is the fuzzy load power. The terms in the first bracket on the right-hand side (RHS) of equation (4.108) can be considered as the fuzzy base load, and it depends only on time, whereas the second bracket contains the temperature-dependent fuzzy load terms.

Equation (4.108) can be rewritten as

$$\begin{aligned}(m_j(t),\, \alpha_j(t)) = &\left\{(p_{0,}\, c_0) + \sum_{i=1}^{n}[(p_i,\, \theta_i)x_i(t) + (b_i,\, \beta_i)y_i(t)]\right\}_j \\ &+ \{(\gamma_0, c_0^{'})T_j(t) + (\gamma_1, c_1)T_j(t-1) + (\gamma_2, c_2)T_j(t-2) \\ &+ (\gamma_3, c_3)T_j(t-3)\}_j, \qquad j = 1, \ldots, m\end{aligned} \tag{4.109}$$

For simplicity, let

$$x_i(t) = \sin i\omega t, \qquad i = 1, \ldots, n \tag{4.110a}$$

$$y_i(t) = \cos i\omega t, \qquad i = 1, \ldots, n \tag{4.110b}$$

In equation (4.109), the first letter in all small brackets of the equations indicates the middle of the parameter, and the second letter indicates the spread of that parameter. A triangular membership is used for each parameter.

In the fuzzy model developed in equation (4.109), the task is to find the fuzzy model parameters to minimize the spread of the output. Mathematically, the fuzzy linear optimization problem can be expressed as

Minimize

$$\begin{aligned}J = \Bigg|\sum_t \Bigg\{\sum_{j=1}^{m} m_j(t) - &\Bigg\{p_0 + \sum_{i=1}^{n}[p_i x_i(t) + b_i y_i(t)]_j + \gamma_0 T_j(t) + \gamma_1 T_j(t-1) \\ &+ \gamma_2 T_j(t-2) + \gamma_3 T_j(t-3)\Bigg\} + \Bigg\{\alpha_j(t) - \Bigg[c_0 + \sum_{i=1}^{n}\Big\{\theta_i x_i(t) + \beta_i y_i(t)\Big\}_j \\ &+ \underline{c}^{'}_0 T_j(t) + c_1 T_j(t-1) + c_2 T_j(t-2) + c_3 T_j(t-3)\Bigg\}\Bigg]\Bigg\}\Bigg|\end{aligned} \tag{4.111}$$

where $t \in [0,\, t_F]$, and t_F is the number of days for which data are taken at the hour in question.

Subject to satisfying the following two constraints for each measurement point given as

$$\begin{aligned} m_j(t) - (1-\lambda)\,\alpha_j(t) \geq & \left\{ p_0 + \sum_{i=1}^{n} \{[p_i x_i(t) + b_i y_i(t)]\}_j \right. \\ & \left. + \gamma_0 T_j(t) + \gamma_1 T_j(t-1) + \gamma_2 T_j(t-2) + \gamma_3 T_j(t-3) \right\} \\ & - \left\{ c_0 + \sum_{i=1}^{n} \{\theta_i x_i(t) + \beta_i y_i(t)\right\}_j + c_0' T_j(t) + c_1 T_j(t-1) \\ & + c_2 T_j(t-2) + c_3 T_j(t-3), \qquad j = 1, \ldots, m \end{aligned} \tag{4.112}$$

$$\begin{aligned} m_j(t) + (1-\lambda)\,\alpha_j(t) \leq & \left\{ p_0 + \sum_{i=1}^{n} [p_i x_i(t) + b_i y_i(t)] + \gamma_0 T(t) + \gamma_1 T_j(t-1) \right. \\ & \left. + \gamma_2 T_j(t-2) + \gamma_3 T_j(t-3) \right\} \\ & + \left\{ c_0 + \sum_{i=1}^{n} \theta_i x_i(t) + \beta_i y_i(t) \right\}_j + c'_0 T_j(t) + c_1 T_j(t-1) \\ & + c_2 T_j(t-2) + c_3 T_j(t-3) \qquad j = 1, \ldots, m \end{aligned} \tag{4.113}$$

The problem formulated in equations (4.111) to (4.113) is one of linear optimization and can be solved by linear programming.

Having obtained the middle and spread of each fuzzy parameters, we can calculate the load power at any hour in question using equation (4.109).

4.6 Conclusion

This chapter presented a new formulation for fuzzy short-term load-forecasting models. In the first part of the chapter, the load power is considered given as crisp (non-fuzzy) data, while the load model parameters are fuzzy, having certain middles and spreads. The problem turns out to be one of linear optimization.

In the second part of the chapter, the load power is considered to be fuzzy power data having certain middles and spreads. Three different fuzzy models—A, B, and C—were developed, and new fuzzy equations were obtained. The resulting optimization problem is linear and can be solved using linear programming.

References

[1] T.J. Ross, Fuzzy Logic with Engineering Applications, McGraw-Hill, New York, 1995.
[2] M.E. El-Hawary, Electric Power Applications of Fuzzy Systems, IEEE Press, New York, 1998.

[3] J. Nazarko, W. Zalewski, An application ofthe fuzzy regression analysis to the electrical load estimation, Electrotechnical Conference MELECON '96, 3, 1563–1566, IEEE Catalog #96CH35884, May 13–16, 1996.
[4] H. Tanaka, S. Uejima, K. Asai, Linear regression analysis with fuzzy model, IEEE Trans. Syst. Man Cybern 12 (6) (1982) 903–907.
[5] P.T. Chang, E.S. Lee, Fuzzy least absolute deviations regression based on the ranking of fuzzy numbers, IEEE World Congr. Fuzzy Syst. IEEE Proc. 2 (1994) 1365–1369.
[6] J. Watada, Y. Yabuchi, Fuzzy robust regression analysis, IEEE World Congr. Fuzzy Syst. IEEE Proc. 2 (1994) 1370–1376.
[7] R. Alex, P.Z. Wang, A new resolution of fuzzy regression analysis, IEEE Int. Conf. Syst. Man Cybern. 2 (1998) 2019–2021.
[8] H. Ishibuchi, M. Nii, Fuzzy regression analysis by neural networks with non-symmetric fuzzy number weights, IEEE Int. Conf. Neural Networks 2 (1996) 1191–1196.
[9] H. Ishibuchi, M. Nii, Fuzzy regression analysis with non-symmetric fuzzy number coefficients and its neural network implementation, Proc. Fifth IEEE Int. Conf. Fuzzy Syst. 1 (1996) 318–324.
[10] S. Ghoshray, Fuzzy linear regression analysis by symmetric triangular fuzzy number coefficients, Proc. IEEE Int. Conf. Intell. Eng. Syst. (1997) 307–313.
[11] M.-S. Yang, C.-M. Ko, On cluster-wise fuzzy regression analysis, IEEE Trans. Syst. Man. Cybern. Part B Cybern. 27 (1) (1997) 1–13.
[12] H. Tanaka, H. Lee, Interval regression analysis by quadratic programming approach, IEEE Trans. Fuzzy Syst. 6 (4) (1998) 473–481.
[13] C.J. Huang, C.E. Lin, C.L. Haung, Fuzzy approach for generator maintenance scheduling, Electr. Power Syst. Res. 24 (1992) 31–38.
[14] I.M. Altas, A.M. Sharaf, A fuzzy logic power tracking controller for a photovoltic energy conversion scheme, Electr. Power Syst. Res. 25 (1992) 227–238.
[15] C.S. Chen, J.N. Sheen, Applying fuzzy mathematics to the evaluation of avoided cost for a load management program, Electr. Power Syst. Res. 26 (1993) 117–125.
[16] N. Rajakovic, S. Ruzic, Sensitivity analysis of an optimal short term hydro-thermal schedule, IEEE Trans. Power Syst. 8 (3) (1993) 1235–1241.
[17] R. Kenarangui, A. Seifi, Fuzzy power flow analysis, Electr. Power Syst. Res. 29 (1994) 105–109.
[18] J.Y. Fan, J.D. McDonald, A real-time implementation of short-term load forecasting for distribution power systems, IEEE Trans. Power Syst. 9 (2) (1994) 988–944.
[19] D. Srinivasan, M.A. Lee, Survey of hybrid fuzzy neural approach to electric load forecasting, IEEE Int. Conf. Syst. Man Cybern. 5 (1995) 4004–4008.