BIALYSTOK TECHNICAL UNIVERSITY

MONOGRAPHS No. 22

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MODELING OF ELECTRICAL POWER DISTRIBUTION SYSTEMS

ISSN 0867-096X

BIALYSTOK TECHNICAL UNIVERSITY PUBLISHER

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Thanks is awarded to the Committee for Scientific Research for its financial donations which contributed towards the making of this book

PRINTED IN POLAND BY BIALYSTOK TECHNICAL UNIVERSITY PUBLISHER

Drukowano w Zakładzie Poligraficznym Politechniki Białostockiej. Nakład 100 egz.
Format B-5. Ark. wyd. 6. Ark. druk. 6,5

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1. INTRODUCTION

Electrical power distribution systems are large-scale dynamic systems changing both in time and area. Constant development of the systems causes the continual increase of the amount of information that is necessary to rational control of system operation. The systems, traditionally used to direct the power distribution system operation, are not adapted for collecting and transforming such a great number of data and they do not allow a dispatcher to utilize the information effectively.

Quite new possibilities are given by the development of microprocessor technology. Electrical power industries are intensively computerized both in Poland and abroad. In Poland this process is connected with the development of information technology in the power engineering industry, power utilities and substations [19, 72, 83, 90, 122]. The basic functions of computer aided systems for power distribution system operation, that actually operate or are prepared to be introduced, are to assist the dispatcher in the field of communication, supervision, control and advisement [54]. The requirements according to the quality of electrical energy supplied to customers and requirements to the system operation are still increasing and it causes that demands of real time control of distribution system operation are going to be current. The problem is very important as electrical power distribution systems make 85% of the power distribution property in the national electric power system. About 80% of energy losses are created in distribution systems and they are the reason of about 90% of all interruptions in the energy supply [46].

To know the power system state vector it is important to define the power system state changes as the dynamic system and the synthesis of the controlling algorithms. However the state vector, by which the power distribution system may be fully characterized, cannot be defined for a technical reason. In such a situation to solve the task of defining a whole power system state vector we must use all other accessible, usually random disturbed, information.

An efficient use of available data is the greatest difficulty in the optimal control of power distribution system. If the analysis of distribution system operation state is performed on the basis of out-of-data or incorrect information, all decisions accepted on its base may significantly differ from optimum decision.

This fact is usually omitted in power distribution system analysis. It is assumed that accuracy of analysis is defined by the adopted method and that the information needed to take a decision may be obtained in the process of the power distribution system operation. Because of this the most methods of the power distribution system operation state analyses [55] cannot be used in practice since the data is unreliable or does not exist at all.

The practical application of the algorithms of analysis and real time control of the power distribution system operation needs such methods of forming and estimating of power distribution system state vector that will be based on the really existing data and will take into consideration the fact that the data may contain different types of errors. The results should give enough accuracy in practical calculations. A good mathematical tool for describing the systems in uncertain conditions is the theory of estimation.

Contrary to power transmission system for which the number of measurements is much bigger in relation to the number necessary for representation of transmission system state, the power distribution systems have a great deficiency of real time measurements. The large part of information on element loads comes only from sporadic measurements carried out in different time. The estimation of the power distribution system state is made on the basis of the formal differentiated information of different accessibility and reliability. The both, transmission and distribution systems have their own specific structures and operational configurations. It implies the necessity of development of separate numerical representation and computational algorithms. Therefore the separate state estimation methods for both types of systems must be worked out and they should take into account the specificity of the systems.

The literature on the theory and application of the power system state estimation is very rich [15, 29, 33, 38, 39, 45, 60, 95, 97, 98, 111, 112], but the literature on the estimation of operation states of power distribution systems is very poor and it is usually limited to formulating and solving partial problems [18, 25, 32, 69, 75, 101, 107, 116, 117].

In the works [69, 107, 116] the possible ways to gain the information about the power distribution system demands were analyzed. The issue of load modeling was often discussed. They suggest to use typical load diagrams and additional information on customers (energy consumption in different time periods, average level of utilization of rated power of a transformer, single measurements of loads). Hypothetical load curves, obtained in this way, are corrected on the basis of power balance in the lines supplying the subclasses of the nodes - the receipt points. The quoted literature does not show exactly the way of estimation of likelihood level of initial evaluation of loads that is then used to correct them. It does not show the way of typical load diagrams construction.

The publication [101] suggests load estimation on the basis of annual energy consumed by the customers who are divided into classes of differentiated load shapes. Customers are divided on classes according to the tariff, the energy consumption, and specificity of customers. For the customers of one class, the annual peak power is defined on the base of empirical dependence as the function of annual energy consumption. A 24-hour load profile of the MV/LV (middle voltage/low voltage) substation and its peak load are defined by adding up all of the twenty-four hours load diagrams of groups of each class of customers.

The similar approach to the subject, where the load of substation is modeled as a sum of component loads of each class of receivers is presented in papers [18, 116].

In Poland the most difficulty about the practical implementation of the method is lack for a well-organized system of measurement of electrical energy customer loads. In some countries (Great Britain, France) measurements and registration of electrical energy consumer loads are made on a larger scale [59, 62, 73, 109]. In Great Britain there is a system of continuous registration of the representative electrical energy users' groups. Typical twenty-four hours load diagrams of each customer are prepared on the basis of the measurement results. Those diagrams are analyzed to define the influence of different factors on the shape of load curves and to qualify the participation of each electrical energy consumers' group in the total load.

In the publications [32, 117] there is a proposition of a peak load estimation at power distribution system buses on the basis of the staff's opinion.

The work [25] proposes the method of Bayessian statistic conclusion to the estimation of the twenty-four hours and annual peak loads of the MV/LV substation. This method lets effectively utilize the formal differentiated information on the load process and the customers.

Although the presented methods cannot be directly applied to the real time estimation of the operation state of the distribution system, they allow to initially transform the primarily accessible information about loads.

The mathematical model and the basic algorithm of the iterative solution of the task of the electric power system state estimation were published in the main publications [95, 97, 98].

The idea of application of the Kalman filter theory [4] to the dynamic estimation of the state vector of electrical power system was published in [60] and [98]. The publications [111, 112] are very important for the development of this study direction. Other results were published in the works [15, 21, 51, 100].

The review of the literature shows that the issue of the power distribution system operation states estimation has not been treated on a broad basis so far. The methods mentioned above are the basis for considerations presented in the book. However before they are utilized, they should be modified and adapted to the power distribution system circumstances.

This book is an effort to solve the problem of the estimation of the electrical power distribution system operation states that is the basis for the real time control of power distribution systems.

The enclosed, at the end of the work, list of references consists of the publications directly connected with the given considerations (it is marked in the text) and those, which gave the author some general ideas necessary to solve the problem.

The book consists of 7 chapters. The first one is the introduction.

The second chapter shows the information structure and defines the basic aims of the computer aided real time control system for the medium voltage electrical power distribution systems.

The third chapter shows the results of the analysis of the effect of unreliable input information on the results of calculations made on the power distribution system. On the basis of the theory of experimental design a qualitative model of a system was constructed and experimentally verified. The chapter presents also the results of a computer simulation experiment that was made to evaluate the influence of errors in determination of the load at the system buses on the results of the distribution system calculations.

The fourth chapter presents the theoretical basis of the methods of the static and dynamic estimation of the system state vector. The basic stochastic properties of the process of the load variations at the system nodes were determined by the analysis of the results of the load measurements. On this basis the two-stage algorithm of the load estimation at the system nodes was proposed. In the first part the initial load diagrams at the system nodes are created using the given primary information. In the second part of the algorithm, the initial load curves are corrected on the basis of the accessible measurements of voltages and power or current flows in the elements of the system. The static estimation method is based on the generalized least squares method and the dynamic estimation method is based on equations of the extended Kalman filter. Algorithms of numerical solutions of estimator equations derived for static and dynamic cases are given.

The fifth chapter shows the results of the simulation and measuring studies which verified the estimation methods proposed in the fourth chapter. All results, both calculations and experiments, refer to the part of the existing 15 kV municipal power distribution system.

The sixth chapter presents the most important, made by the author, computer programs for numerical analysis of experimental data and the programs for the electrical power distribution system calculations.

The last chapter, seventh, presents the main conclusions.

2. POWER DISTRIBUTION SYSTEM AS AN CONTROLLED OBJECT

2.1. Functions of the system

The system analysis, as a study method [42], assumes the representation of the studied object as a system which transforms the reaction of the environment (input variables) on the object into the system answer (output variables). Electrical power distribution systems are included into so called self-organizing system class. Within the structure of such a system two basis subsystems, differ because of their functions, are noticed - the control subsystem (dispatcher control and automatic control equipment) and the controlled object (a set of functionally connected devices and material resources).

The computer methods of control and the algorithms of their realization need the information system structure to have additional, functionally differentiated subsystems (Fig. 2.1):

- subsystem of observation and transmission of the information,
- subsystem of estimation of the object state,
- forecasting subsystem,
- optimization subsystem.

Input signals may, in a general case, pass both into an object (for example the customer demand for electric power and energy, the voltage level in the feeding system, the random factors) and into a subsystem of control (as above, and planned tasks, the instructions from higher dispatcher levels).

An output signal is created by an object as a result of technological and organizational processes that proceed in the object (the level of the realization of power and energy demands, the quality of the electrical energy, technological parameters of energy transmission and distribution, the level of the realization of planned tasks and instructions).

So in every time moment the system realizes the following projection [42]:

$$f = \varphi \cdot h : \mathbf{V} \times \mathbf{U} \to \mathbf{X} \to \mathbf{Y} \qquad (2.1)$$

In the formula:

V - the set of the feasible input values,

U - the set of the feasible controls,

X - the set of the feasible system states,
Y - the set of the feasible output values,
h - the projection transforming input and admissible controls set into the set of the admissible states:

$$\mathbf{h}: \mathbf{V} \times \mathbf{U} \rightarrow \mathbf{X} \tag{2.2}$$

Where: φ – the projection transforming the set of the admissible system states into the set of the admissible output values:

$$\varphi: \mathbf{X} \rightarrow \mathbf{Y} \tag{2.3}$$

The form of projection f, which describes the electric power system operation in relation to the environment, depends on the specific control $u(t) \in U$. Each self-organizing system has its own aim of operation which describes how the control subsystem effects on the object. The control is made by the appropriate choice of the control parameter values, which may be changed by the control subsystem (the change of controller and protections adjustments, reconnections in the network, plans and corrections of plans). The control should be chosen in such a way to ensure the possible most efficient system operation in considered time horizon. To this end the control subsystem should include the quality function of the control:

$$\mathbf{c}: \mathbf{V} \times \mathbf{U} \times \mathbf{X} \times \mathbf{Y} \rightarrow \mathbf{P} \tag{2.4}$$

and the relation

$$\mathbf{R} \subset \mathbf{P} \times \mathbf{P} \tag{2.5}$$

which linearly orders all quality indicators of the system operation.
In the formulae:
c - the function of the quality of control,
P - the set of the indicators of the quality of the system operation.

Under such formulation the determination of the required distribution system control is the optimization problem under the uncertain conditions. The uncertainty appears at input, at output and in the nature of the system itself and it is caused by:
- the random character of the customers' demand for electrical power and energy,
- the influence of the random factors on the system,

- fuzzy character of the functions c, h and the relation $\mathbf{R}$, caused by the absence of the full formalization of aims of the system and by the presence of informal intuitive procedures in the decision process,
- fuzzy character of the sets $\mathbf{X}$, $\mathbf{U}$, $\mathbf{V}$ caused by the uncertainty of corresponding a priori information.

The object state and its answer may be observed with the help of the appropriate set of measuring devices, called the observation subsystem. The signals may be observed (measured) directly or together with the disturbance.

The measuring devices give their own errors, both random and systematic, due to the structure of devices or with the measuring principle. Generally, the measurements do not give the sufficient information about the object state and they may be inadequate to estimate system operation.

Generally, the state estimation is the utilization of the disturbed observations to evaluate the properties of the real system.

The decisions, taken by the control system, may be received by the controlled object practically only some time after their taking. The time delay is caused by the loss of time for the transmission of measurements, calculations and the transmission of the control signals to the controlled object. Generally, the control worked out for the given time interval is not optimal in the next time interval. Furthermore, after some time delay the control may be impossible for execution because of the different values of limitations than at the time when the decision was taken. Therefore procedures of the real time control of system operation should include subsystems of the forecasting of time series which will make those procedures effective and stable. The basis of control choice is the forecast of values of time series (for example loads at system nodes) in the next time interval, obtained from the statistical information and accessible observations in the specified moment.

The control of the system operation consists in defining the way of control signals' change as to force the desired manner of the object operation.

If the given quality indicator is used to estimate the behavior of the object (for example power and energy losses in the system, the quality of energy supplied to the customers) and the control value is defined so as to minimize or maximize the value of this indicator, the problem is called the optimal control.

The above problem may be written mathematically as follows [42], [71]:

$$
\begin{aligned}
&c(\hat{x}(t+\Delta t),\ u(t+\Delta t),\ \hat{y}(t+\Delta t),\ \hat{v}(t+\Delta t)) \rightarrow \text{extr} \\
&\hat{x}(t+\Delta t) \in \mathbf{X},\ \ u(t+\Delta t) \in \mathbf{U}, \\
&\hat{y}(t+\Delta t) \in \mathbf{Y},\ \ \hat{v}(t+\Delta t) \in \mathbf{V}
\end{aligned}
\tag{2.6}
$$

where:

$\hat{x}(\cdot)$ – the estimate of the system state vector,

$u(\cdot)$ – the control vector,

$\hat{y}(\cdot)$ – the estimate of output vector,

$\hat{v}(\cdot)$ – the estimate of input vector,

t – the moment of the last observations accessible for the control subsystem,

Δt – the time-delay between the moment t and the moment when the controlled object receives the control signals.

Fig. 2.1 shows the diagram of the information structure of the real time control system of the operation of electrical power distribution systems.

2.2. Real time control

As the home power distribution systems develop and are provided with telecontrol and automatic devices and as computers are introduced in regional load-dispatching units or directly at the system substations, the problems of the real time control of the distribution system operation must be solved.

The control in real time is to compensate the inner and exterior disturbances and to provide the desirable conditions of the system operation.

The conception of real time refers to the efficiency of processing of physical process in the time of its actual run and it shows that the results of processing may be utilized for the control of the process.

Time and mode of the calculations linked with the real time control of electrical power distribution system should fulfill the time of control operation realization. It will be the time of decimal part of second (for the instantaneous tripping), of several or dozen minutes (for load ratio control) and of several or dozen hours (for network reconnections).

In this section the main tasks of the computer system of real time control of operation of electrical power distribution systems of medium voltage are described [58].

Solution to the presented below problems is the basis to work out a computer system of the real time control of electric power distribution system operation.

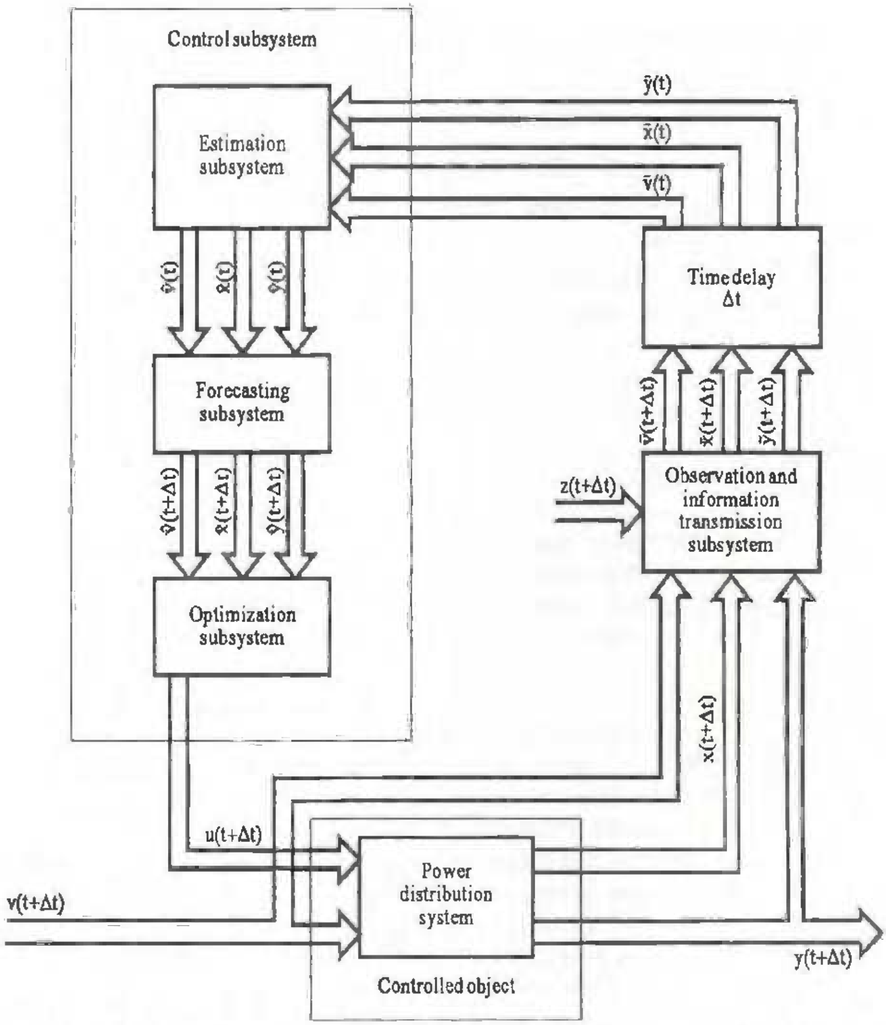


Fig. 2.1. The diagram of the information structure of the real time control system of the operation of electrical power distribution systems.

Denotations:

$v(t + \Delta t)$ - the value of the system input at the moment $t + \Delta t$,

$x(t + \Delta t)$ - the value of the system state at the moment $t + \Delta t$,

$y(t + \Delta t)$ - the value of the system output at the moment $t + \Delta t$,

$z(t + \Delta t)$ - the random disturbance of the observation,

$\tilde{v}(t)$ - the observed input value of the system at the moment t,

$\tilde{x}(t)$ - the observed state value of the system at the moment t,

$\tilde{y}(t)$ - the observed output value of the system at the moment t,

$\hat{v}(t)$ - the estimate of input value of the system at the moment t,

$\hat{x}(t)$ - the estimate of state value of the system at the moment t,

$\hat{y}(t)$ - the estimate of output value of the system at the moment t,

$\hat{v}(t+\Delta t)$ - forecast of the system input value, made at the moment t for the moment $t+\Delta t$,

$\hat{x}(t+\Delta t)$ - forecast of the system state value, made at the moment t for the moment $t+\Delta t$,

$\hat{y}(t+\Delta t)$ - forecast of the system output value, made at the moment t for $t+\Delta t$ moment,

Δt - the time-delay between the moment t and the moment when the controlled object receives control signals.

Telemetring processing and control

The real possibilities of installing the telemetring transmitters at the electric power HV/MV substations let assume that the telemetry will include the following quantities:

- the flow of active and reactive power in transformers at HV/MV substations,
- the currents in MV lines outgoing from HV/MV substations,
- the voltages at HV and MV busbars at HV/MV substations.

Before the results of telemetry are utilized in computational algorithms the control of their reliability is necessary.

The basis for the examination of the measurements reliability is the balance of currents and power. As usually the telemetry of current modules are mainly given, the current balance, according to the first Kirchhoff law, may be satisfied only approximately. The analysis of the load characteristic of distribution systems shows that the difference between the geometric and algebraic sums of currents is not bigger than 5%. If the unbalance exceeds the given value, it is necessary to look for the incorrect measurement. To do it the complementary methods of the control of the telemetry reliability are used [45, 58, 68].

The levels of voltage at the HV and MV busbars of HV/MV substation are an important parameter for many tasks of the control of the system operation. To define the reliability of voltage telemetring, it is necessary to use the verified methods of control. The most efficient one concerns on comprising of the values of measurements given by the measuring systems allocated at busbars of different voltage.

The control of load of the system elements and of the voltage levels is done directly at the places of the measurement while the data is being collected and processed.

The control and the identification of the system configuration

The control may be both sporadic or periodic. It depends on the kind of telecontrol devices allocated to the system. In the first case, when the signals about the change of the configuration do not appear, the last system configuration saved in the computer memory is accepted as the really existing. Only when the loss of accordance between the information about the connectors position and the telemetry appears, the control is done. In the second case the signals verifying the position of the controlled connectors are sent periodically every the same time intervals.

All changes of the distribution system configurations should be automatically registered and noticed whether the change had been caused by the dispatcher operation or by the operation of automatics.

In the task it should be also checked whether the system, which is to be radial, is devoid of loop and whether it is coherent.

Power systems state estimation

The estimation is the utilization of the disturbed observations to evaluate the real system properties. The application of the estimation theory to estimate the system state vector lets extend the efficiency of control of the power distribution system operation. If the estimation of the system state can be stated with the satisfactory precision, the necessary control is often obvious or it can be obtained with the deterministic theory of control [96].

The short-time load forecasting

To efficiently operate the power distribution systems, it is necessary to have the reliable information on the power demand in some advance (from several minutes to tens of hours)

The forecast process may be divided into the following stages:

- collecting and preparing the input data (statistical information),
- processing of statistical data in order to determine the unknown model parameters and dependence joining the object characteristic, liable to forecast, with time and known variables,
- calculation of the forecast for the given moment and of the given value of different known variables.

The procedure of the short-time forecasting of the load should be adaptive, i.e. it should join the identification of the model parameters with their estimation [16].

The optimization of system operational configurations

As the providing of power distribution systems in breakers by remote control and adopting the remote control functions by microprocessor systems development, the efficient optimization of operation configurations of the power distribution systems will be possible. The optimization of configuration is to reduce power and energy losses in lines and transformers.

The extensive review of design methods of operation distribution system configurations is placed in [81].

The optimization of voltage regulation

The voltage deviations at receiver terminals from optimum values are the reasons of economic loss suffered by customers. Nowadays the programming of optimum voltage regulation is made only for the representative days and system configuration [55].

There is a technical possibility of permanent voltage regulation at HV/MV transformers with the load ratio control. The operation of the regulator of transformer voltage consists in keeping the set voltage level at MV busbars of HV/MV substation for the given time intervals. If the circuit of current compensation is in service the voltage level at MV buses in the particular time interval is the linear function of the transformer load. That does not ensure the optimal voltage level at the heterogeneous load diagrams at the system buses.

The real time state estimation and the short-time load prediction techniques will make possible the on-line voltage optimization in the distribution systems supplied from the HV/MV substations.

The optimization of the operation program of the compensation devices

The proper management of the reactive power effects on the reduction of the power losses, the increase of the transmission capacity and the supporting of the proper voltage levels at the system buses.

The proper providing of the distribution systems in the telecontrol devices will make the on-line control of the operation of the reactive power compensation devices possible.

The load control

It is necessary to turn customers off from the electric supply while the system emergency states or the great power deficit. It can be realized by the microcomputer installed at the supplied substation or at local dispatcher center. In such a case the list, order, and conditions of the possible load disconnection for each supplying substation should be included to the input information set.

The microcomputer executes the instructions about the necessary load shedding which are received from the computer of the higher dispatcher level or from the dispatcher. It chooses the lines to switch off, the order of the load disconnection and controls the energizing lines. It also warns the customers - thorough the dispatcher or directly - about the possible load disconnection. It is possible to increase the power demand by the voltage reduction at the supplying bus. It is necessary here to know the load-voltage characteristics [79].

The choice of the protection settings and the verification of the system configurations considering the rated thermal current of the devices

When the configuration of the controlled distribution system or the scheme of the supplying substation are changed it is necessary to recalculate the short-circuit currents and to verify the correctness of the choice of the protection settings and the feasibility of the system configuration considering the rated thermal currents of the devices in the short-circuit conditions.

If the reconnections in the distribution system are made without interruption of the customer supply it is necessary to close for the short time the chosen loops at the system that causes the flow of the circulating currents. The real current flow at the particular line sections is obtained as the result of the superposition of the component and circulating currents. The calculations of the circulating currents make possible the verification of the possibility of reconnections with the load switches.

The fault localization

The service of power distribution systems in the fault conditions is one of the basic factors which conditions the continuity of the customers supply. The economic losses arising during the searching of the faulted section of the electric line play an important role in the total economic losses suffered by the electric power customers because of the interruption of supply. It is necessary to find the faulted section to make reserve reconnections or make the pointwise (exact) localization of the damage.

The utilization of microcomputers to real time control of the system operation will it make possible to reduce the time of the fault localization and to limit the inconvenience for the customers results of disturbance [76].

3. THE EFFECT OF UNRELIABLE INPUT INFORMATION ON THE RESULT OF ANALYSIS OF DISTRIBUTION SYSTEMS

3.1. Available information

Real time control of power distribution systems requires the acquisition and processing of a large amount of data on system parameters and operating conditions.

This section describes sets of information that are actually available on the level of local power distribution utility [82].

System structure, configuration and parameters of its components

Data on system structure and parameters of its components belongs to a group of stable data. In this case there is no problem in completing proper data at appropriate departments of a power utility. Such departments regularly bring up to date technical documentation. Usually there is no discrepancy between real and documentation state.

Configuration of power distribution system changes all the time. Changes in system configuration are recorded in a operational log-book and then visualized on a proper panel in dispatching unit. Because of that and a large amount of information, calculating schemes often become out-of-date even before their preparation.

Traditional methods of the tracing of system configuration changes are ineffective. An important advantage of computer systems is the possibility of real time mapping of system configuration changes. Data on system configuration changes can be formed on the base of the telesignaling of switches' positions and of dispatcher commands. Recording of the time when change occurs and type of change allows retrospective calculations and analysis of system states.

Measurements

An attendant at the power utility dispatcher center makes an every hour load statistic that concerns selected 220 kV and 110 kV busbars and transmission lines which connect the power utility with other utilities and with power stations.

The load statistic includes values of active and reactive power flows and values of voltage levels in 220 kV and 110 kV systems. Data is collected from substations with a permanent twenty-four hours service. On this basis an attendant at the dispatcher center prepares a power balance for the utility. Readings of loads and voltages on the middle voltage level (MV) are also provided for selected substations of 220 kV/110 kV/MV.

Readings of loads of 110 kV/MV transformers (active and reactive power and voltage) and MV feeders (current) at every 110 kV/MV substation are provided every Wednesday at the evening load peak time.

A special measuring day (usually on Wednesday) is organized twice a year (in July and December). Power flow in 220 kV, 110 kV, 30 kV, 20 kV, 15 kV and 6 kV is checked at 3.00 AM, 11.00 AM and at the peak load time. Loads of transformers are also checked. The voltage level in middle voltage system is maintained by transformer voltage regulators according to a chart given by utility dispatcher center. The voltage level in 220 kV and 110 kV systems depends on the situation in the transmission system.

All 220 kV and 110 kV lines are provided with active and reactive energy meters, and 110 kV/MV transformers are provided with loss meters. Some of MV lines are provided not only with ammeters but also with wattmeters.

According to Polish regulations [94], measurements of line loads and voltage levels on the beginning and the end of lines should be made at least:

- once a shift at the substations with permanent twenty-four hours service,
- once a year at the 110 kV and higher substations without permanent twenty-four hours service (possible at the peak load time),
- every five years at the other substations.

A monthly energy balance is made on the level of a region power utility. Balance energy losses in HV system, determined from meter indications, and estimated balance losses in MV and LV systems (total) are given in a report.

The commerce department prepares a report on monthly energy sale. This report shows the energy sale to different customer groups with different tariffs estimated on the basis of periodical bills.

The report is aided by two computer settlement systems: "Zbyt" for individual customer and "AWO" for industrial customers [85].

Data on customers

The basic documentation containing data on customers is the energy meters reading book and account book. Secondary documentation consists of the reports on energy sale.

Different tariffs are applied in a settlement of accounts of electrical energy and power delivery to different classes of customers [24]. The type of tariff gives information about customer type and measurement equipment.

For small households and commercial customers available information is:
- type of customer,
- annual active energy consumption.

For industrial customers available information depends on measurement equipment. The minimal set of information is:
- type of customer (branch of production, single-shift or multi-shift work, type of receivers),
- monthly active energy consumption.

Also the following data may be available:
- monthly reactive energy consumption,
- monthly 15-minutes peak load,
- load curve (coming from hourly readings of energy meters).

3.2. Input data errors evaluation

The quality of an information model of a power distribution system may be characterized by calculation speed and accuracy of results.

Calculation speed depends mainly on the type of computer and efficiency of computation algorithms. Accuracy depends mainly on mathematical model used, errors in equivalent circuits of system components and errors of observations.

An investigation of input data errors and their influence on system analysis has a particular importance in real time modeling of power distribution systems.

Mathematical model

A load flow in a radial system is a function of system connections, components parameters, bus loads, and voltage at the supplying bus [3]. The following components of distribution systems are usually included in an equivalent circuit for steady-state calculations:
- overhead and cable lines,
- transformers,
- feeder reactors,
- shunt capacitors,
- connectors.

Distribution system components are modeled by their equivalent circuits in terms of resistance, reactance, conductance, and susceptance. Under balanced conditions a distribution system can be represented by a single phase model. Feeder reactors, shunt capacitors and connectors are usually modeled by two-terminal network. Overhead and cable lines are represented by π-equivalent circuits. Transformers are modeled by Γ-equivalent circuits. The components interconnections constitute the equivalent circuit of a distribution system. Loads are normally specified by their constant active and reactive power requirement assuming they are unaffected by small variations of voltage and frequency during normal steady-state operation.

Calculations and analysis for distribution systems are made based on the models constructed this way. They have satisfactory accuracy and the errors caused by assumed representation are practically insignificant [3, 6]. To ensure required precision of the load flow, calculations are made in an iterative way [9]. The main difficulty in operational practice is the accurate determination of parameters and the loads at receiving buses.

An error in the determination of the overhead line resistance R_L primarily results from omission of an influence of weather conditions, load current, quality of materials, and also from inaccuracy in stating of real wires length and cross-sectional area. Errors in determination of reactance X_L and susceptance B_L of overhead lines mainly result from constructional deviations and from differences between real and computational wires length and radius.

Parameters R_C, X_C, G_C, and B_C of equivalent circuits of cable lines are usually determined from available per-kilometer data. Per-kilometer data are average values for specified types of cables. Per-kilometer data are multiplied by the cable length to obtain values of calculated parameters. Both variable conditions of cables manufacture and cabling, and variable conditions of their operating cause that real values of cable parameters are different from average values.

Errors in calculated values of resistance R_T, reactance X_T, conductance G_T, and susceptance B_T of equivalent circuits of transformers come primarily from transformer ratio, open-circuit losses, impedance losses, open-circuit current, short-circuit voltage tolerance values. Changes in the temperature of transformer winding have an influence on changes of winding resistance and effect errors in determination of R_T parameter.

Ranges of possible errors in determination of values of parameters of equivalent circuits of main system components according to [48] are shown in Table 3.1

Table 3.1

Ranges of possible errors in determination of values of parameters of equivalent circuits

Parameter	Possible error range
R_L	±16%
X_L, B_L	±3%
R_K	±10%
X_K,B_k	±3%
R_T	±22%
X_T, B_T,G_T	±10%

Load at system receiving buses

A proper evaluation of loads is an essential point in correct calculations and analysis of power distribution systems. The acquisition of this data is complex because of a large number of nodes and their area distribution. As a rule receiving nodes are not equipped with stationary measuring instruments so measurements of loads are performed sporadically.

In operational practice in Poland [32, 57], load data for the performance calculations and analysis of distribution systems are usually acquired from the power utility operational staff. The utility staff evaluate loads on the basis of possessed knowledge on results of sporadic (usually annual) measurements of receiving transformers' peak loads and data on customers supplied from transformers. These assessments contain different types of errors. These errors with their effects on system analysis results have a particular impact in real-time modeling of power distribution systems. Proper evaluation of loads at system buses is an essential part of system analysis.

In order to evaluate the quality of load evaluation by the power utility operational staff a special experiment was designed and carried out. For an existing municipal 15 kV distribution system the measurement of daily peak load was taken at 26 randomly selected 15/0.4 kV receiving transformers. At the same time the utility staff were asked to evaluate the loads. Seven dispatchers from the local power utility took part in the experiment. The results of measurements and dispatcher evaluations are shown in Table 3.2.

Table 3.2

Evaluation of substation peak load by utility staff

No.	Name of substation	Transformer rating	Measured value of peak load	Evaluation of substation peak load						
				Dispatcher number						
				1	2	3	4	5	6	7
-	-	kVA	kW	kW						
1	PS 29	50	34.4	15	30	40	40	50	30	40
2	PS 64	160	20.4	10	-	50	20	20	-	25
3	PS 67	160	95.4	115	95	100	140	100	100	140
4	PS 151	160	118.6	64	80	110	35	55	80	30
5	PS 170	315	67.6	60	135	250	30	40	150	40
6	PS 173	160	29.6	80	85	110	15	30	90	20
7	PS 234	400	172.0	130	-	200	-	100	-	-
8	PS 261	200	64.2	38	85	160	15	40	150	30
9	PS 268	500	312.0	195	185	450	190	380	200	150
10	PS 288	400	94.0	80	75	160	50	80	100	70
11	PS 315	400	123.0	103	160	450	45	80	100	60
12	PS 377	400	132.6	97	150	160	55	60	160	50
13	PS 426	400	163.0	51	150	225	48	60	180	50
14	PS 471	250	183.0	103	155	250	50	60	175	50
15	PS 485	250	54.8	38	160	210	45	70	100	50
16	PS 550	75	74.4	35	55	210	30	50	60	30
17	PS 629	160	111.0	109	75	130	75	-	95	70
18	PS 651	160	96.4	110	65	60	30	45	80	25
19	PS 746	160	109.0	95	60	110	65	80	80	70
20	PS 765	200	77.4	71	80	110	35	60	100	40
21	PS 855	630	35.9	45	150	100	22	50	45	20
22	PS 1136	40	42.8	9	-	140	-	15	-	-
23	PS 1195	250	59.4	23	170	130	15	-	86	20
24	PS 1237	250	147.2	121	150	110	40	80	150	50
25	PS 1243	100	145.0	77	80	80	35	40	60	30
26	PS 1326	400	157.0	98	150	200	30	70	170	35

The dispatcher evaluations were compared with measurement results. On this basis percentage errors of expert judgments were calculated as given by

$$\delta_{Pi} = \frac{P_{ie} - P_{im}}{P_{im}} \times 100\% \qquad (3,1)$$

where:

P_{ie} - dispatcher evaluation of peak load at bus i,

P_{im} - measured value of peak load at bus i.

The results of error calculation are presented in Table 3.3.

Investigations referred to an active load because the dispatchers were not able to estimate a reactive load.

Initial analysis of data shows that errors in a peak load evaluation made by the dispatcher are contained in the range from -80% to +320%. Similar variation ranges were taken for a reactive load.

Voltage at a supplying bus

Uncertainty in determination of voltage at a supplying bus is caused by errors brought by voltage transformer (1% - 3%), measuring instrument (1.5% - 2.5%) and telemetry (2.5%).

3.3. The effect of unreliable input information on the result of analysis of power distribution systems

The investigation of the effect of unreliable input information on the result of analysis of power distribution systems may be divided into two main parts.

In the first part, verification of significance of input quantities is made. This is a basis for the eventual elimination from the qualitative model those input quantities that may be recognized as insignificant in assumed variation interval of all input quantities. The significant input quantities are placed in the sequence from the most significant to the least significant.

Elimination investigations are made only for verification of the significance of the influence of input quantities on output quantities without determination of the form functional dependency between them [88].

Table 3.3

Percentage errors (δ_p) of utility staff evaluations of a peak load

No.	Name of substation	Percentage error						
		Dispatcher number						
		1	2	3	4	5	6	7
-	-	%						
1	PS 29	-56.4	-12.8	16.3	16.3	45.3	-12.8	16.3
2	PS 64	51.0	-	145.1	2.0	2.0	-	22.5
3	PS 67	20.5	-0.4	4.8	46.8	4.8	4.8	46.8
4	PS 151	-46.0	-32.5	-7.3	-70.5	-53.6	-32.5	-74.7
5	PS 170	-11.2	-99.7	269.8	-55.6	-40.8	121.9	-40.8
6	PS 173	170.3	187.2	271.6	-49.3	1.4	-32.4	-32.4
7	PS 234	-24.4	-	16.3	-	-41.9	-	-
8	PS 261	-40.8	32.4	149.2	-76.6	-37.7	133.6	-53.3
9	PS 268	-37.5	-40.7	44.2	-39.1	21.8	-35.9	-51.9
10	PS 288	-14.9	-20.2	70.0	-46.8	-14.9	6.4	-25.5
11	PS 315	-16.3	30.1	82.9	63.4	-35.0	-18.7	-51.2
12	PS 377	-26.8	13.1	86.6	-58.5	-54.8	20.7	-62.3
13	PS 426	-68.7	-8.0	28.8	-70.6	-63.2	10.4	-69.3
14	PS 471	-43.7	-15.3	14.8	-72.7	-67.2	-4.4	-72.7
15	PS 485	-30.7	192.0	137.2	-17.9	27.7	82.5	-8.8
16	PS 550	-53.0	-26.1	-19.4	-59.7	-32.8	-19.4	-59.7
17	PS 629	-1.8	-32.4	-0.9	-32.4	-	-14.4	-36.9
18	PS 651	14.1	-32.4	14.1	-68.9	-53.3	-17.0	-74.1
19	PS 746	-12.8	-45.0	-8.3	-40.4	-26.6	-26.6	-35.8
20	PS 765	-8.3	3.4	80.9	-54.8	-22.5	29.2	-48.3
21	PS 855	25.3	317.8	262.1	-38.7	39.3	25.3	-44.3
22	PS 1136	-79.0	-	-41.6	-	-65.0	-	-
23	PS 1195	-61.3	186.2	118.9	-74.7	-	44.8	-66.3
24	PS 1237	-17.8	1.9	-25.3	-72.8	-45.7	1.9	-66.0
25	PS 1243	-46.9	-44.8	-44.8	-75.9	-72.4	-58.6	-79.3
26	PS 1326	-37.6	-4.5	27.4	-80.9	-55.4	8.3	-77.7

In the second part probabilistic characteristics of investigated output quantities are determined. Random input quantities are characterized by probability density functions. A convenient way to estimate probabilistic characteristic of output quantities is experimental simulation [30, 86].

On initial stage of analysis and optimization of distribution system operating conditions, possible large number of quantities that characterize system and have influence on investigated output characteristic should be taken into account. Elimination of some quantities without sufficient justification but only for fear that a model might be too large is a mistake that may shatter the sense of all work when omitted quantities have a significant influence on investigated system characteristic.

In order to verify the significance of investigated factors on analyzed output quantity, the theory of experimental design is applied [43, 64, 65, 74, 88]. Elimination investigations are made in accordance with special schemes of experiment design. The result of this investigation is experimentally verified a qualitative model of power distribution system.

The design of experiment is a procedure of selection of the number and conditions of realizations of experiments that are necessary and sufficient for solution of a stated problem with required accuracy as well as the procedure of selection of a mathematical method of analysis of experiment results and a decision criterion [88].

A scheme of the experiment is a set of values of investigated quantities determined in accordance with principles of the theory of experimental design for which sets of values of output quantities are determined by methods and measurements adequate for the investigated object [88].

The following sets are defined [88].

Set of input quantities

$$\mathbf{X}_C, \quad \{X_k : k = 1,2,\ldots,i\}, \tag{3.2}$$

where i is a number of input quantities.

Set of output quantities

$$\mathbf{Z}, \quad \{Z_p : p = 1,2,\ldots,w\}, \tag{3.3}$$

where w is a number of output quantities.

Set of constants

$$\mathbf{C}, \quad \{C_l : l = 1,2,\ldots,s\}, \tag{3.4}$$

where s is a number of constants.

A qualitative mathematical model of an investigated object is introduced in the following form:

$$F(X_1, X_2, ..., X_i, Z_1, Z_2, ..., Z_w) = 0 \tag{3.5}$$

Then the investigated object is decomposed on w formal objects and each of them is described by only one output quantity. This leads to the succeeding relations:

$$\begin{gathered} F_1(X_1, X_2, ..., X_i, Z_1) = 0, \\ F_2(X_1, X_2, ..., X_i, Z_2) = 0, \\ \cdot \\ \cdot \\ \cdot \\ F_w(X_1, X_2, ..., X_i, Z_w) = 0. \end{gathered} \tag{3.6}$$

The above decomposition is used to define the response function of the investigated object:

$$Z_p = F(X_1, X_2, ..., X_i) \tag{3.7}$$

The response function approximates the results of observations. These results are sets of values of an output quantity that correspond to specific arrangements of input quantities. They are determined in the following way:

$$\mathbf{Z}, \quad \{Z_p^{(u)} : p = 1, 2, ..., w,\ u = 1, 2, ..., n\}. \tag{3.8}$$

A scheme of an experimental design is made by using the set of n arrangements of input quantities:

$$X_c, \quad \{Z_{k/u} : k = 1, 2, ..., i,\ u = 1, 2, ..., n\}, \tag{3.9}$$

where:

i - number of input quantities,

n - number of arrangements of input quantities in investigated scheme.

The number of arrangements depends on an accepted scheme of the experiment. In elimination investigations with the greater number of investigated quantities (approximately $i > 5$) fractional double level schemes play a particular role [43, 88]. In such plans the number of values of investigated quantities is limited to two ($n_k = 2$). Then it is aimed to decrease a number of arrangements compared to a full scheme of the experiment in such a way that there is a possibility to determine an optimal approximated linear polynomial.

In order to simplify a procedure investigated quantities are normalized to [-1, 1] range. Relation for normalization is described by the following form:

$$\hat{X}_k = \frac{X_k - \overline{X}_k}{\Delta X_k}, \qquad (3.10)$$

where:

$$\overline{X}_k = \frac{X_{k\,min} + X_{k\,max}}{2} \qquad (3.11)$$

$$\Delta X_k = \frac{X_{k\max} - X_{k\min}}{2}, \qquad (3.12)$$

where:

$\hat{X}_k$ - normalized value of investigated quantity X_k,

$X_{k\,min}, X_{k\,max}$ - minimal and maximal value of investigated quantity X_k,

$\overline{X}_k$ - average value of investigated quantity X_k,

ΔX_k - variability range of investigated quantity X_k.

Inverse relation:

$$X_k = \overline{X}_k + \hat{X}_k \Delta X_k. \qquad (3.13)$$

Because of the reconnaissance character of investigations it is allowed to approximate the response function by linear model:

$$Z_p = b_{p,0} + b_{p,1}X_1 + b_{p,2}X_2 + \ldots + b_{p,i}X_i \qquad (3.14)$$

where: $b_{p,0}, b_{p,1}, b_{p,2}, \ldots, b_{p,i}$ are polynomial coefficients.

The number of unknown coefficients is

$$N_b = i + 1. \qquad (3.15)$$

Unknown polynomial coefficients are calculated on the basis of the solution of the general regression equation [62]. In the case of fractional double level scheme with normalized variables it leads to the following form:

$$b_{p,k} = \frac{1}{n} \sum_{u=1}^{n} \hat{X}_{k/u} Z_p^{(u)} \tag{3.16}$$

where:

$$\hat{X}_0 = 1$$

denotes the hypothetical variable.

Verification of the significance of polynomial coefficients is usually made by means of the t-Student test [52]. In order to attain this it is necessary to calculate t statistics for each coefficient b_k of polynomial (3.16)

$$t(b_{p,k}) = \frac{|b_{p,k}|}{S(b_{p,k})}, \quad k = 0,1,\ldots,i \tag{3.17}$$

where: $S(b_{p,k})$ - standard deviation of $b_{p,k}$ coefficient.

Verification of significance is performed by comparing the significance level α_k of each statistic $t(b_{p,k})$ to the assumed critical value α.

It allows us to order investigated quantities in accordance with increasing values of their significance levels. Because values of significance levels α_k are a quantitative measure of significance of their influence on investigated output quantity it has a great importance in further merit analysis.

The assumption of a given value of significance level α_k allows us to distinguish investigated input quantities as significant if

$$\alpha_k \leq \alpha, \tag{3.18}$$

and insignificant if

$$\alpha_k > \alpha, \tag{3.19}$$

In the case of a fractional double level plan standard deviations of polynomial coefficients are the same and they are equal to

$$S(b_{p,k}) = \frac{S(Z_p)}{\sqrt{n}} \tag{3.20}$$

where:

$S(b_{p,k})$ - variation of output quantity,

n - a number of investigated input quantities in an accomplished scheme of experiment.

In the case of electrical power distribution systems, the experiment is usually made using computers on numerical model (computer experiment). In such a situation, variations of single measurements are equal to zero and variation $S(Z_p)$ has to be calculated in indirect way.

Relationship for evaluation of a variation of output quantity is determined on the basis of the theorem of moments of a linear function of random variables [8]. If investigated quantities X_k are not correlated random variables then the expected value of output quantity Z_p given by the equation (3.14) is equal to

$$E[Z_p] = \sum_{k=0}^{i} b_{p,k} E[X_k] \tag{3.21}$$

and variation

$$Var[Z_p] = \sum_{k=0}^{i} b_{p,k}^2 Var[X_k] \tag{3.22}$$

In the case of a double level scheme of experiment with normalized investigated quantities they may be considered as random variables with expected values

$$E\left[\hat{X}_0\right] = 1, \tag{3.23}$$

$$E\left[\hat{X}_k\right] = 0 \quad k = 1, 2, \ldots, h$$

If the random variable X_k has a normal distribution, then its values with the probability equals to 0.9973 are placed in the following interval

$$E[X_k] - 3\sigma_k < X_k < E[X_k] + 3\sigma_k \tag{3.24}$$

For normalized investigated quantities the following condition is satisfied

$$-1 \le \hat{X}_k \le 1 \quad k = 1, 2, \ldots, i. \tag{3.25}$$

Taking into account equations (3.23) - (3.25) standard variations of normalized investigated quantities can be estimated as follow:

$$S(\hat{X}_0) = 0 \tag{3.26}$$

$$S(\hat{X}_k) \approx \frac{1}{3} \quad k = 1, 2, \ldots, i. \tag{3.27}$$

In relation to the above equations the evaluation of expected values of output quantity Z_p is

$$\overline{Z}_p = b_{p,0} \tag{3.28}$$

and variation

$$S^2(Z_p) = \frac{1}{9}\sum_{k=1}^{i} b_{p,k}^2 \tag{3.29}$$

Verification of the significance of investigated quantities in accordance with the double level saturated scheme is made under the assumption that it is accepted, in an assumed range of variability of investigated quantities, to approximate function of investigated object by a linear model. Experimental verification of permission to use the linear model consists in including additional, central arrangement of values of investigated quantities that correspond with normalized values equal to 0 to the experiment scheme.

In order to evaluate nonlinearity the t-Student test is used. The equation for this test is

$$\left| Z_{p}\big|_{\hat{X}_{k-0}} - b_{p,0} \right| < t_{\alpha} S(Z_p) \qquad (3.30)$$

where:

$Z_{p}\big|_{\hat{X}_{k-0}}$ - value of output quantity for central arrangement of investigated quantities,

$b_{p,0}$ - constant coefficient of a linear model,

$S(Z_p)$ - standard deviation of output quantity,

t_{α} - critical value of t-Student test, determined for the significance level α and $f = n - 1$ degrees of freedom.

If the condition (3.30) is satisfied it means that the linear model is adequate not only for boundary values but also for their arithmetic averages. Thus there is a great probability that the linear function is adequate also for other intermediate values.

On the basis of physical principles of energy transmission and distribution process, 15 basic input quantities have been chosen for investigation. They represent parameters of transformers and lines, supplying bus voltage, peak active and reactive load, and power factor:

X_1 - resistance of supplying transformer, r_t,
X_2 - reactance of supplying transformer, x_t,
X_3 - conductance of supplying transformer, g_t,
X_4 - susceptance of supplying transformer, b_t,
X_5 - sum of resistance of receiving transformers, Σr_{to},
X_6 - sum of reactance of receiving transformers, Σx_{to},
X_7 - sum of conductance of receiving transformers, Σg_{to},
X_8 - sum of susceptance of receiving transformers, Σb_{to},
X_9 - sum of resistance of lines, Σr_l,
X_{10} - sum of reactance of lines, Σx_l,
X_{11} - sum of susceptance of lines, Σb_l,
X_{12} - supplying bus voltage, u,
X_{13} - receiving transformers transformation ratio, ϑ,
X_{14} - sum of peak active loads of receiving transformers, ΣP,
X_{15} - power factor, $tg\varphi$.

The system components are described by the basic functions as: power flow, power and energy losses, and voltage level. The following 9 basic global indices characterizing operating conditions of power distribution systems have been chosen as output quantities for further analysis:

Z_1 - active power at supplying bus, P,
Z_2 - reactive power at supplying bus, Q,
Z_3 - current amplitude in supplying transformer, I,
Z_4 - total active power losses, ΔP,
Z_5 - total load losses, ΔP_o,
Z_6 - total no-load losses, ΔP_j,
Z_7 - daily active energy consuming by system, A,
Z_8 - daily active energy losses, ΔA,
Z_9 - sum of square voltage deviation at the receiving buses, $\Sigma \delta U^2$.

Because of the reconnaissance character of investigations it is allowed to approximate the response function by the linear model:

$$Z_p = b_{b,0} + b_{b,1} X_1 + b_{b,2} X_2 + \ldots + b_{b,15} X_{15} \qquad p = 1, 2, \ldots, 9 . \tag{3.31}$$

In order to eliminate insignificant quantities from the qualitative model of a power distribution system, a special scheme of experiment was worked out. The scheme is constructed on the basis of the Plackett-Burman algorithm [43]. The assumed scheme needs the following number of arrangements

$$n = 2^{15-11} + 1 = 17 . \tag{3.32}$$

The number of arrangements for the full double level scheme would be

$$n_k = 2^{15} = 32768 . \tag{3.33}$$

The simulation experiment was carried out for a numerical model of an existing 15 kV, radially operated municipal power distribution system (ref. Appendix). The system is supplied from 110 kV/65 kV, 16 MVA supplying transformer and supplies 65 receiving transformers of 15/0.4 kV.

Values of investigated quantities and their range of variability are shown in Table 3.4.

Table 3.4.

Values of investigated quantities

k	Investigated quantity	$X_{k\,min}$	$X_{k\,max}$	X_k	ΔX_k	X_k		
						-1	0	1
1	r_t	$0.8\ r_{tn}$	$1.2\ r_{tn}$	$1.0\ r_{tn}$	$0.2\ r_{tn}$	0.071 Ω	0.089 Ω	0.107 Ω
2	x_t	$0.9\ x_{tn}$	$1.1\ x_{tn}$	$1.0\ x_{tn}$	$0.1\ x_{tn}$	1.533 Ω	1.703 Ω	1.873 Ω
3	g_t	$0.9\ g_{tn}$	$1.1\ g_{tn}$	$1.0\ g_{tn}$	$0.1\ g_{tn}$	58.68 mS	65.2 mS	71.72 mS
4	b_t	$0.9\ b_{tn}$	$1.1\ b_{tn}$	$1.0\ b_{tn}$	$0.1\ b_{tn}$	283.5 mS	315 mS	346.5 mS
5	Σr_{to}	$0.8\ \Sigma r_{ton}$	$1.2\ \Sigma r_{ton}$	$1.0\ \Sigma r_{ton}$	$0.2\ \Sigma r_{ton}$	1856 Ω	2320 Ω	2784 Ω
6	Σx_{to}	$0.9\ \Sigma x_{ton}$	$1.1\ \Sigma x_{ton}$	$1.0\ \Sigma x_{ton}$	$0.1\ \Sigma x_{ton}$	3650.4 Ω	4056 Ω	4461.6 Ω
7	Σg_{to}	$0.9\ \Sigma g_{ton}$	$1.1\ \Sigma g_{ton}$	$1.0\ \Sigma g_{ton}$	$0.1\ \Sigma g_{ton}$	252.2 mS	280.2 mS	308.2 mS
8	Σb_{to}	$0.9\ \Sigma b_{ton}$	$1.1\ \Sigma b_{ton}$	$1.0\ \Sigma b_{ton}$	$0.1\ \Sigma b_{ton}$	1571.4 mS	1746 mS	1920.6 mS
9	Σr_l	$0.88\ \Sigma r_l$	$1.12\ \Sigma r_{ln}$	$1.0\ \Sigma r_{ln}$	$0.12\ \Sigma r_{ln}$	23.03 Ω	26.17 Ω	29.31 Ω
10	Σx_l	$0.97\ \Sigma x_l$	$1.03\ \Sigma x_{ln}$	$1.0\ \Sigma x_{ln}$	$0.03\ \Sigma x_{ln}$	10.92 Ω	11.26 Ω	11.60 Ω
11	Σb_l	$0.97\ \Sigma b_l$	$1.03\ \Sigma b_{ln}$	$1.0\ \Sigma b_{ln}$	$0.03\ \Sigma b_{ln}$	2415.3 mS	2490 mS	2564.7 mS
12	u	$0.95\ u_n$	$1.1\ u_n$	$1.025\ u_n$	$0.075\ u_n$	14.25 kV	15.375 kV	16.50 kV
13	ϑ	$0.9\ \vartheta_n$	$1.05\ \vartheta_n$	$0.975\ \vartheta_n$	$0.075\ \vartheta_n$	35.44	38.39	41.34
14	ΣP	$0.4\ \Sigma S_n$	$0.6\ \Sigma S_n$	$0.50\ \Sigma S_n$	$0.10\ \Sigma S_n$	6890.0 kW	8612.5 kW	10335 kW
15	tg φ	0.18	0.42	0.30	0.12	0.18	0.30	0.42

Symbols with subscript n mean rated values of particular quantities

Table 3.5.

Scheme of experiment and results of calculation of output quantities

Arrangements	Investigated quantities																Output quantities								
	X_0	r_t	x_t	g_t	b_t	Σr_{to}	Σx_{to}	Σg_{to}	Σb_{to}	Σr_l	Σx_l	Σb_l	u	ϑ	ΣP	tg φ	P	Q	I	ΔP	ΔP_0	ΔP_l	A	ΔA	$\Sigma\delta U^2$
-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	kW	kvar	A	kW	kW	kW	kWh	kWh	$(\%)^2$
1	+	+	+	+	+	–	+	–	+	+	–	–	+	–	–	–	6850	1579	242	177.1	80.2	96.9	108592	3284	1491
2	+	+	+	+	–	+	–	+	+	–	–	+	–	–	–	+	6861	3396	299	188.9	128.2	60.7	108288	2980	323
3	+	+	+	–	+	–	+	+	–	–	+	–	–	–	+	+	10330	5637	499	320.9	253.6	67.3	162556	4595	273
4	+	+	–	+	–	+	+	–	–	+	–	–	–	+	+	+	10436	5392	454	426.8	372.1	54.7	163637	5676	756
5	+	–	+	–	+	+	–	–	+	–	–	–	+	+	+	+	10302	5183	388	293.1	232.0	61.1	162181	4219	236
6	+	+	–	+	+	–	–	+	–	–	–	+	+	+	+	–	10255	2445	362	245.5	179.9	65.6	161659	3697	297
7	+	–	+	+	–	–	+	–	–	–	+	+	+	+	–	+	6832	2996	255	158.8	82.8	76.1	108122	2814	321
8	+	+	+	–	–	+	–	–	–	+	+	+	+	–	+	–	10295	2560	363	285.6	207.4	78.2	162293	4331	1421
9	+	+	–	–	+	–	–	–	+	+	+	+	–	+	–	+	6854	3234	298	181.5	135.6	45.9	108006	2699	653
10	+	–	–	+	–	–	–	+	+	+	+	–	+	–	+	+	10284	5123	389	274.5	193.3	81.2	162199	4238	1390
11	+	–	+	–	–	–	+	+	+	+	–	+	–	+	+	–	10316	3084	421	307.1	254.1	53.0	162224	4262	687
12	+	+	–	–	–	+	+	+	+	–	+	–	+	+	–	–	6843	1416	242	170.5	98.0	72.5	108219	2912	320
13	+	–	–	–	+	+	+	+	–	+	–	+	+	–	–	+	6862	3051	258	189.3	97.0	92.3	108689	3381	1441
14	+	–	–	+	+	+	+	–	+	–	+	+	–	–	+	–	10316	2841	422	307.0	236.3	70.7	162461	4499	305
15	+	–	+	+	+	+	–	+	–	+	+	–	–	+	–	–	6853	1655	280	180.3	131.5	48.7	108040	2732	648
16	+	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	6819	1569	279	146.5	87.6	58.9	107764	2456	386
17	+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	8567	3129	333	226.2	159.7	66.4	135123	3488	302

Table 3.6.

Polynomial coefficients and corresponding values of t-Student statistic and significance levels

Number	Output quantity	Investigated quantities																S(z)
		b_o	Σr_t	x_t	g_t	b_t	Σr_{to}	Σx_{to}	Σg_{to}	Σb_{to}	Σr_l	Σx_l	Σb_l	u	ϑ	ΣP	tg φ	
1	P	8581.70	8.80	-1.90	4.10	-4.00	14.30	-6.30	16.40	-3.50	12.00	-5.90	-7.90	-16.40	4.60	1735.00	13.40	578.4
	t	59.34	0.06	0.01	0.03	0.03	0.10	0.04	0.11	0.02	0.08	0.04	0.05	0.11	0.03	12.00	0.09	
	α	0.0	0.95	0.99	0.98	0.98	0.92	0.97	0.91	0.98	0.94	0.97	0.96	0.91	0.98	0.0	0.93	
2	Q	3197.60	9.80	63.70	-19.20	5.60	-10.80	28.30	51.90	34.40	12.20	-14.80	-246.70	-153.40	-21.20	835.60	1053.90	459.9
	t	27.81	0.09	0.55	0.17	0.05	0.09	0.24	0.45	0.30	0.11	0.13	2.14	1.33	0.19	7.27	9.17	
	α	0.0	0.93	0.60	0.87	0.96	0.93	0.81	0.66	0.77	0.91	0.90	0.05	0.20	0.85	0.0	0.0	
3	I	337.50	1.10	-0.40	0.30	-0.20	0.70	-0.10	5.30	0.10	0.60	-0.30	-2.80	-25.20	-0.10	68.40	11.20	24.7
	t	54.71	0.17	0.07	0.05	0.03	0.11	0.01	0.86	0.01	0.09	0.05	0.46	4.08	0.01	11.09	1.81	
	α	0.0	0.87	0.95	0.96	0.98	0.91	0.99	0.40	0.99	0.93	0.96	0.67	0.001	0.99	0.0	0.09	
4	ΔP	240.80	8.80	-1.90	4.00	-4.00	14.40	-6.20	16.40	-3.40	11.90	-6.00	-7.90	-16.50	4.60	66.70	13.40	25.4
	t	37.94	1.38	0.29	0.63	0.63	2.26	0.98	2.58	0.53	1.88	0.94	1.24	2.61	0.73	10.51	2.11	
	α	0.0	0.18	0.78	0.54	0.54	0.04	0.38	0.02	0.60	0.08	0.37	0.32	0.02	0.48	0.0	0.05	
5	ΔP_o	173.10	8.90	-1.90	2.40	-4.80	14.70	-6.10	11.20	-3.40	16.80	-5.80	-7.90	-26.80	12.60	68.00	13.70	26.7
	t	25.97	1.31	0.28	0.37	0.73	2.21	0.92	1.67	0.51	1.62	0.87	1.19	4.02	1.90	10.20	2.06	
	α	0.0	0.31	0.78	0.72	0.48	0.04	0.39	0.12	0.62	0.12	0.40	0.33	0.001	0.08	0.0	0.06	
6	A	67.70	0.0	0.0	1.60	0.80	-0.40	-0.10	5.20	0.0	1.10	-0.20	0.0	10.30	-8.00	-1.30	-0.30	4.7
	t	57.04	0.0	0.0	1.34	0.69	0.32	0.06	4.38	0.0	0.95	0.14	0.0	8.63	6.77	11.06	0.27	
	α	0.0	1.0	1.0	0.20	0.50	0.76	0.95	0.001	1.0	0.38	0.89	1.0	0.0	0.0	0.36	0.79	
7	ΔA	135308	98.10	-21.10	66.60	-35.10	168.90	-73.90	254.40	-36.90	151.90	-71.10	-90.30	-63.90	-47.10	27093.10	151.60	9032.2
	t	59.92	0.04	0.01	0.03	0.02	0.07	0.03	0.11	0.02	0.07	0.03	0.04	0.03	0.02	12.00	0.07	
	α	0.0	0.97	0.93	0.98	0.98	0.95	0.98	0.91	0.98	0.95	0.98	0.97	0.98	0.98	0.0	0.95	
8	ΔA	3673.4	98.30	-21.30	66.60	-35.20	167.80	-73.80	254.40	-36.80	151.90	-70.90	-90.60	-63.90	-47.10	766.20	151.20	292.2
	t	50.29	1.35	0.29	0.91	0.48	2.30	1.01	3.48	0.50	2.08	0.97	1.24	0.87	0.64	10.49	2.08	
	α	0.0	0.20	0.78	0.39	0.64	0.04	0.37	0.005	0.63	0.06	0.38	0.32	0.40	0.53	0.0	0.06	
9	$\Sigma\delta U^2$	341.30	-1.80	0.0	-0.90	1.50	-11.00	-2.60	-2.80	-0.60	33.60	-0.1	4.80	523.40	-537.5	-31.40	-19.40	250.6
	t	5.40	0.03	0.0	0.01	0.02	0.18	0.04	0.04	0.01	0.54	0.0	0.07	8.35	8.57	0.50	0.31	
	α	0.0	0.98	1.0	0.99	0.98	0.86	0.97	0.97	0.99	0.60	1.0	0.95	0.0	0.0	0.63	0.76	

Critical value of t-Student test for $f = 15$ freedom degrees and significance level $a = 0.05$ equals to $t_{0.05} = 2.132$

The scheme of the experiment and results of computations are shown in Table 3.5. Sign '+' means the maximum value and '-' the minimum value of investigated quantity. The central arrangements with normalized values '0' are added to the scheme to verify permissibility of using the linear model (3.31).

For practical application of the method of investigation of significance of input quantities influence on the result of system calculations, the special computer program was worked on.

For each arrangement of input quantities the system calculations were performed. Computations were made under the following algorithm:

1. Power flow study and calculations of output quantities ($Z_1,\dots, Z_9$) for each arrangement u of input quantities ($X_1,\dots, X_{15}$).
2. Calculations of polynomial coefficients for each output quantity, according to the formula (3.16).
3. Calculation of variances of output quantities according to the formula (3.29).
4. Verification of permission to use the linear model according to the formula (3.30).
5. Verification of the significance of polynomial coefficients using t-Student test according to the formula (11).

The results of investigation of the significance of influence of input quantities on output quantities are shown in Table 3.6.

The experiment shows that in an assumed range of input quantities variation it is allowed to use a linear model for approximation of A response function for all investigated output quantities.

On the basis of a statistical analysis of calculations, input quantities were divided into two groups: significant and insignificant. In the Table 3.7 significant input quantities for each output quantity, at the level of significance $\alpha = 0.05$, are presented. The input quantities are placed in a sequence from the most significant to the least significant.

As shown, active load at system buses is the most significant factor in most computations made on power distribution systems, such as power flows (P, Q, I), total active power losses (ΔP), total load losses (ΔP_o), energy consumption (A), and energy losses (ΔA). In the assumed range of input quantities variation this factor may be, whereas assented insignificant in computations of no-load losses (ΔP_j) and sum of square voltage deviation at the receiving buses ($\Sigma \delta U^2$).

The results of the experiment show that the most important point in power distribution system calculations and analysis is a proper evaluation of active loads at system nodes.

Table 3.7

Significant input quantities (α=0.05)

Output quantity	Permissibility of linear model	Significant input quantities
P	+	ΣP
Q	+	$tg\varphi$, ΣP, Σb_l
I	+	ΣP, u
ΔP	+	ΣP, u, Σg_{to}
ΔP_o	+	ΣP, u, Σr_{to}
ΔP_j	+	u, υ, Σg_{to}
A	+	ΣP
ΔA	+	ΣP, Σg_{to}, Σt_{to}
$\Sigma \delta U^2$	+	υ, u

This problem is a subject investigations in detail in the next sections of this book.

3.4. Simulation studies

In order to properly analyze the performance of distribution circuits, it is essential to obtain accurate estimates of power consumption. The deficiency of measured data to be used in the load models is often very apparent, especially in distribution systems. In order to express the uncertainty in such models, statistical methods are applied.

As shown in [80], analytical description of basic parameters of probabilistic models for power flow and power losses are difficult. It needs the knowledge of many characteristics describing stochastic processes occurring in a system and needs many simplifications.

One of the methods that enables us to avoid these difficulties is computer simulation. Computer simulation is useful today and very often an irreplaceable research and design tool [30, 86].

As mentioned above, in operational practice in Poland, load data for the performance calculations and analysis of distribution systems are acquired from the power utility staff. Power utility staff evaluate loads on the basis of possessed knowledge on results of sporadic measurements and data on customers. Those assessments are loaded by errors which influence the accuracy of the system analysis.

Proper evaluation of the influence of errors made in load estimation needs the knowledge of a probability density function for those errors. In order to choose an adequate function a special experiment described in section 3.2 was designed and carried out.

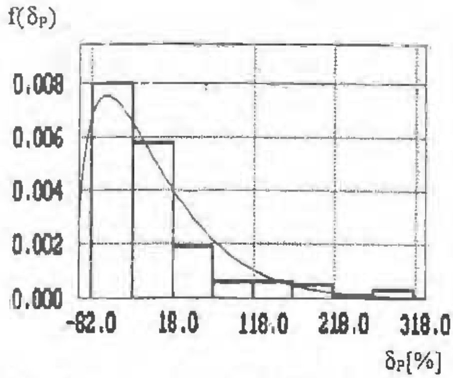


Fig. 3.1. Histogram and the probability density function of relative errors in peak load estimation made by utility staff

In Figure 3.1 a histogram of observed percentage errors, prepared on the basis on Table 3.3, are shown. The sample is characterized by the following parameters:

mean value $\overline{X} = -2.87$, variance $S_x^2 = 5383.30$, standard deviation: $S_x = 73.37$, variance of the mean value $S_{\overline{x}}^2 = 31.67$, standard deviation of the mean value $S_{\overline{x}} = 5.63$, skewness coefficient $a_s = 4.01$, flattening coefficient e = 7.36

The above results indicate that a probability density function for the errors made by utility staff in peak load evaluation is asymmetrical and unimodal. A generalized beta distribution was chosen to represent the experimental data [63].

A random variable has a generalized beta distribution BT(a, b, p, q) when its probability density function is given by the formula

$$f(x) = \begin{cases} \dfrac{1}{B(p,q)} \dfrac{(x-a)^{p-1}(b-x)^{q-1}}{(b-a)^{p+q+1}} & a \le x \le b, p > 0, q > 0 \\ 0 & x < a, x > b \end{cases} \tag{3.34}$$

The normalized constant is a beta function which may be expressed trough gamma functions

$$B(p,q) = \frac{\Gamma(p)\,\Gamma(q)}{\Gamma(p+q)}, \tag{3.35}$$

where

$$\Gamma(\alpha) = \int_0^\alpha x^{\alpha-1} e^{-x} dx. \qquad (3.36)$$

Estimation of parameters of generalized beta distribution may be carried out by moments method [8]:

$$p = \frac{m-a}{\mu_2 (b-a)}\left[(m-a)(b-m) - \mu_2\right], \qquad (3.37)$$

$$q = \frac{b-\overline{x}}{\mu_2 (b-a)}\left[(m-a)(b-m) - \mu_2\right], \qquad (3.38)$$

where:

m – expected value

μ_2 – central moment of second rank (variance).

Limits a and b of the variation interval are determined on the basis of the analysis of possible boundary values of error in load evaluation

$$\delta_{Pi} = \frac{P_{ie} - P_{im}}{P_{im}} \times 100\% \qquad (3.39)$$

where:

P_{ie} - dispatcher evaluation of peak load at bus i,

P_{im} - measured value of peak load at bus i.

On the basis of the analysis of a capacity factor for 15 kV/0.4 kV transformers in municipal distribution systems [78] the following boundary values of transformer load were assumed

$$0{,}1\,S_{ni} \le P_i \le 1{,}0\,S_{ni}, \tag{3.40}$$

where:

P_i - peak active load of transformer i,

S_{ni} - rated load of transformer i.

Then:

$$a = \delta_{p\,min} = \frac{0{,}1\,S_n - S_n}{S_n} \cdot 100\% = -90\%, \tag{3.41}$$

$$b = \delta_{p\,max} = \frac{S_n - 0{,}1\,S_n}{0{,}1\,S_n} \cdot 100\% = 900\%. \tag{3.42}$$

The hypothesis H_0: m = 0 was verified on the significance level $\alpha = 0.05$ [11]. At this assumption the estimator of a second rank central moment was calculated:

$$\hat{\mu}_2 = S_x^2 + \bar{x}^2 \tag{3.43}$$

$$\hat{\mu}_2 = 5391{,}60. \tag{3.44}$$

From equations (3.37) and (3.38) p = 1.275 and q = 12.75 were obtained. Then variable δ_p has a hypothetical beta distribution BT(-90, 900, 1.275, 12.75) and its probability density function is

$$f(\delta_p) = \begin{cases} \dfrac{1}{B(1{,}275,\ 12{,}75)} \dfrac{(x+90)^{1{,}275-1}\,(900-x)^{12{,}75-1}}{900^{1{,}275+12{,}75+1}} & -90 \le x \le 900 \\ 0 & x < -90,\ x > 900 \end{cases} \tag{3.45}$$

The probabilistic density function for BT(-90, 900, 1.275, 12.75) distribution is shown in the same figure as the histogram of observed errors (Figure 3.1).

The result of the approximation (3.45) was confirmed by using a Chi-square test [11]. It was confirmed that on significance level $\alpha = 0.05$ there was no reason for rejection of the hypothesis that theoretical distribution BT(-90, 900, 1.275, 12.75) is a proper model for errors made by utility staff in load evaluation.

Table 3.8.

Estimation of parameters and results of test of goodness of normal distribution fit with errors distribution

Output quantity	Distribution parameters and critical value of significance level	Investigated object								
		Transformer TR1	Line L1	Line L11	Line L29	Line L39	Line L40	Line L60	Line L77	Line L93
I	m	-1.34	0.63	-2.09	-2.17	-0.23	-1.44	-2.29	-1.09	-1.13
	δ	7.42	21.31	13.46	26.84	40.37	13.56	11.76	19.34	36.25
	α	1.00	1.00	0.54	0.05	0.12	1.00	1.00	1.00	0.24
P	m	-1.31	0.64	-2.03	-2.19	-0.27	-1.42	-2.27	-1.09	-1.13
	δ	7.31	21.13	13.24	27.26	38.94	13.37	11.64	19.28	35.56
	α	1.00	1.00	1.00	0.06	0.14	1.00	1.00	1.00	0.26
Q	m	-3.35	0.61	-2.60	-1.96	-0.15	-1.62	-2.54	-1.17	-1.24
	δ	19.13	23.16	15.87	22.34	74.01	15.52	16.83	20.64	39.23
	α	1.00	0.48	0.52	0.23	0.11	1.00	1.00	0.25	0.24
ΔP	m	-1.78	0.81	-2.66	-1.70	1.07	-1.48	-3.21	-0.53	-0.60
	δ	8.73	21.04	19.07	27.33	54.93	15.04	17.38	18.86	49.08
	α	1.00	1.00	0.12	0.00	0.00	1.00	1.00	0.30	0.00
ΔPL	m	-2.87	1.53	-3.45	-2.69	1.04	-1.95	-4.01	0.92	-2.43
	δ	15.42	45.98	33.74	57.62	86.26	37.93	29.19	60.37	70.54
	α	1.00	0.18	0.19	0.00	0.00	0.28	0.28	0.18	0.00
ΔPT	m	-1.20	0.74	-1.67	-1.62	1.08	-1.35	-2.90	-6.70	0.23
	δ	6.45	18.92	13.95	25.08	43.02	11.85	14.62	15.53	41.39
	α	1.00	1.00	1.00	0.00	0.00	1.00	1.00	0.32	0.00
A	m	-1.31	0.10	-1.98	-2.19	-0.28	-1.50	-2.05	-1.15	-1.07
	δ	8.39	19.56	13.83	25.75	38.87	14.29	16.21	19.13	35.57
	α	1.00	1.00	0.35	0.06	0.14	1.00	1.00	1.00	0.29
ΔA	m	-1.23	-0.03	-2.11	-0.98	1.01	-1.19	-2.29	0.53	0.10
	δ	7.67	11.35	16.32	14.27	52.83	14.34	15.39	16.11	33.62
	α	1.00	1.00	0.13	0.00	0.00	0.23	1.00	0.11	0.00

In order to investigate the effect of unreliable evaluation of loads on receiving buses on the results of system computations a special computer simulation experiment was designed and carried out. The goal of the experiment was to determine forms and parameters of probability density function for possible errors in the calculations of basic quantities that characterize operating conditions of a power distribution system. The following output quantities were chosen for analysis:

- active power at the supplying bus at load peak time,
- reactive power at the supplying bus at load peak time,
- current amplitude in the supplying transformer,
- active power flows in a particular MV line at load peak time,
- reactive power flows in a particular MV line at load peak time,
- current amplitude flows in a particular MV line at load peak time,
- total active power losses in the system at load peak time,
- load active power losses in the system at load peak time,
- total active power losses in the part of a system supplied by a particular MV line at load peak time,
- load active power losses in the part of a system supplied by a particular MV line at load peak time,
- daily active energy flows in the system,
- daily energy losses,
- deviations of voltages on receiving buses,
- sum of voltage square deviations on receiving buses.

The simulation process consists of multiple repeated computations of power flows, bus voltages, voltage drops, power and energy losses for randomly changed values of peak loads on receiving buses:

$$P_i = \overline{P}_i \, (1+\delta_{pi}/100) \qquad (3.48)$$

$$Q_i = \overline{P} \, (1+\delta_{pi}/100) \, tg\overline{\varphi}_i \, , \qquad (3.49)$$

where:

P_i – random value of active and reactive peak load on bus i, respectively,

Q_i – random value of the percentage error of load evaluation:

$f(\delta) = BT(-90, 900, 1{,}275, 12{,}75)$,

$\overline{P}_i$ – average value of active peak load on bus i,

$tg\overline{\varphi}_i$ – average value of power factor on bus i.

Simulation calculations are made according to the flow chart below.

1. Generate for the selected bus i a random number δ_{Pi} of BT(-90, 900, 1,275, 12,75) distribution.
2. Calculate P and Q according to the formulae (3.48) and (3.49).
3. Check whether the value P is included in the interval

$$0.1S_{ni} \le P_i \le S_{ni}$$

If so, go to the step 4, and if not, come back to the step 1.

4. Check, if all buses (i = n) were taken into account. If not, choose the next node i + 1 and to go to the step 1, and if so, go to the step 5.
5. Calculate the power flow in the system.
6. Calculate the values of other functions, described for the system branches and nodes.
7. Write down the results of the calculations.
8. Check if the required number of simulations was done. If not, choose the bus i = 1 and go to the step 1, and if so, finish the calculations.

The simulation experiment was carried out for a numerical model of an existing 15 kV, radially operated power distribution system (Appendix 1). According to the results presented above the generalized beta distribution (3.45) was used as a probabilistic model of errors of load evaluations.

In order to calculate system operation states for different bus loads, the special computer program was worked out.

The calculated system was represented in the computer memory as a topological oriented graph. Flow calculations were done by iterative method until desired bus voltage precision was obtained. In the program $\varepsilon = 0.2$ was assumed. Computations were repeated for an assumed number of $n_s = 250$ runs. In each run, new values of peak loads on buses were selected according to equations (3.48) and (3.49)

As a result, n_s realizations of a vector of loads on system buses were obtained

$$P^k = [P_1^k, P_2^k, \ldots, P_i^k, \ldots, P_n^k]^T \qquad (3.50)$$

where:

k - index for realization of simulation,

i - index for receiving bus,

n - number of receiving buses.

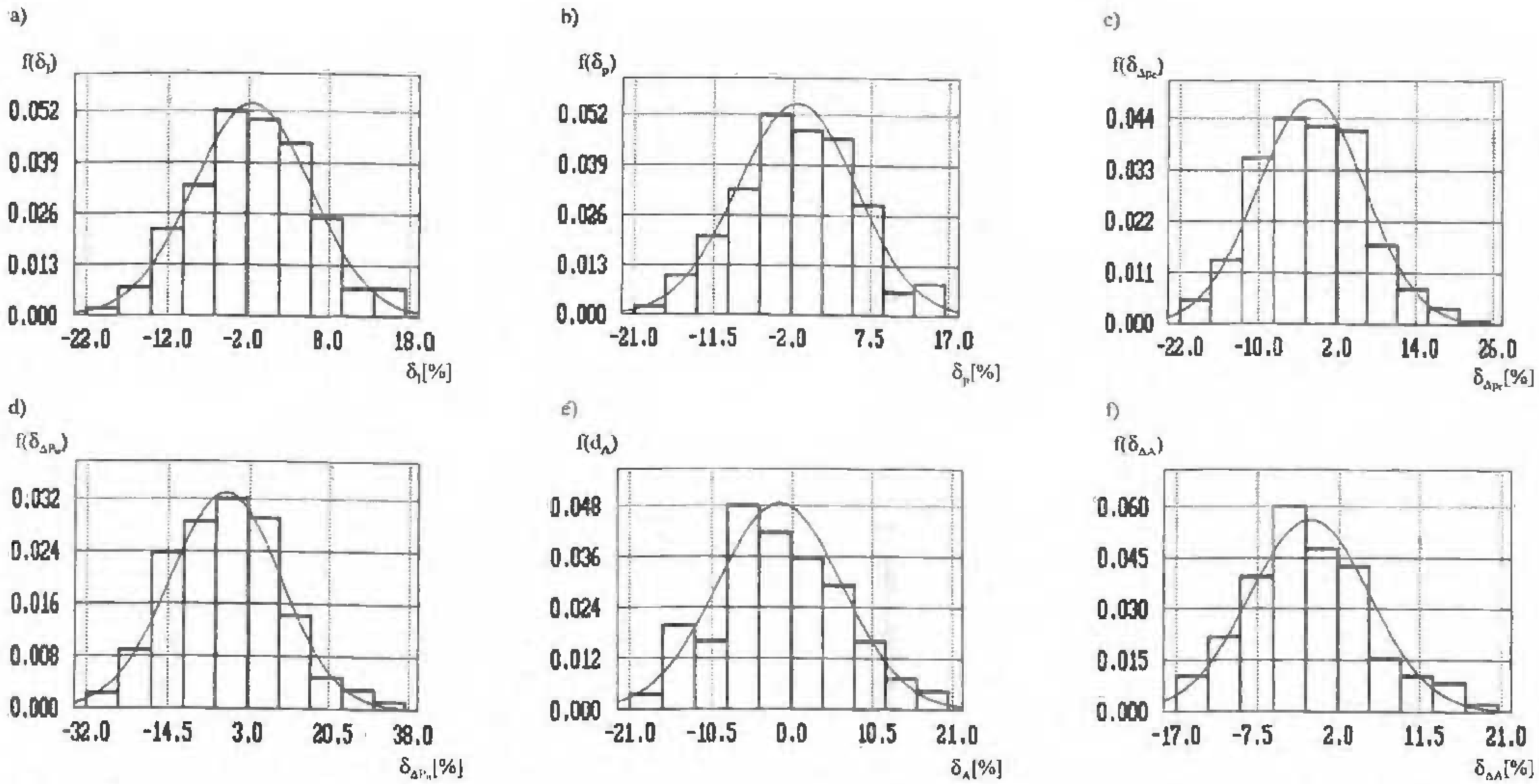


Fig. 3.2. Histograms and probability density functions for percentage errors of calculations: a) current amplitude in supplying transformer, b) active power at supplying bus, c) total active power losses, d) total load losses, e) daily energy consumption, f) daily active energy losses

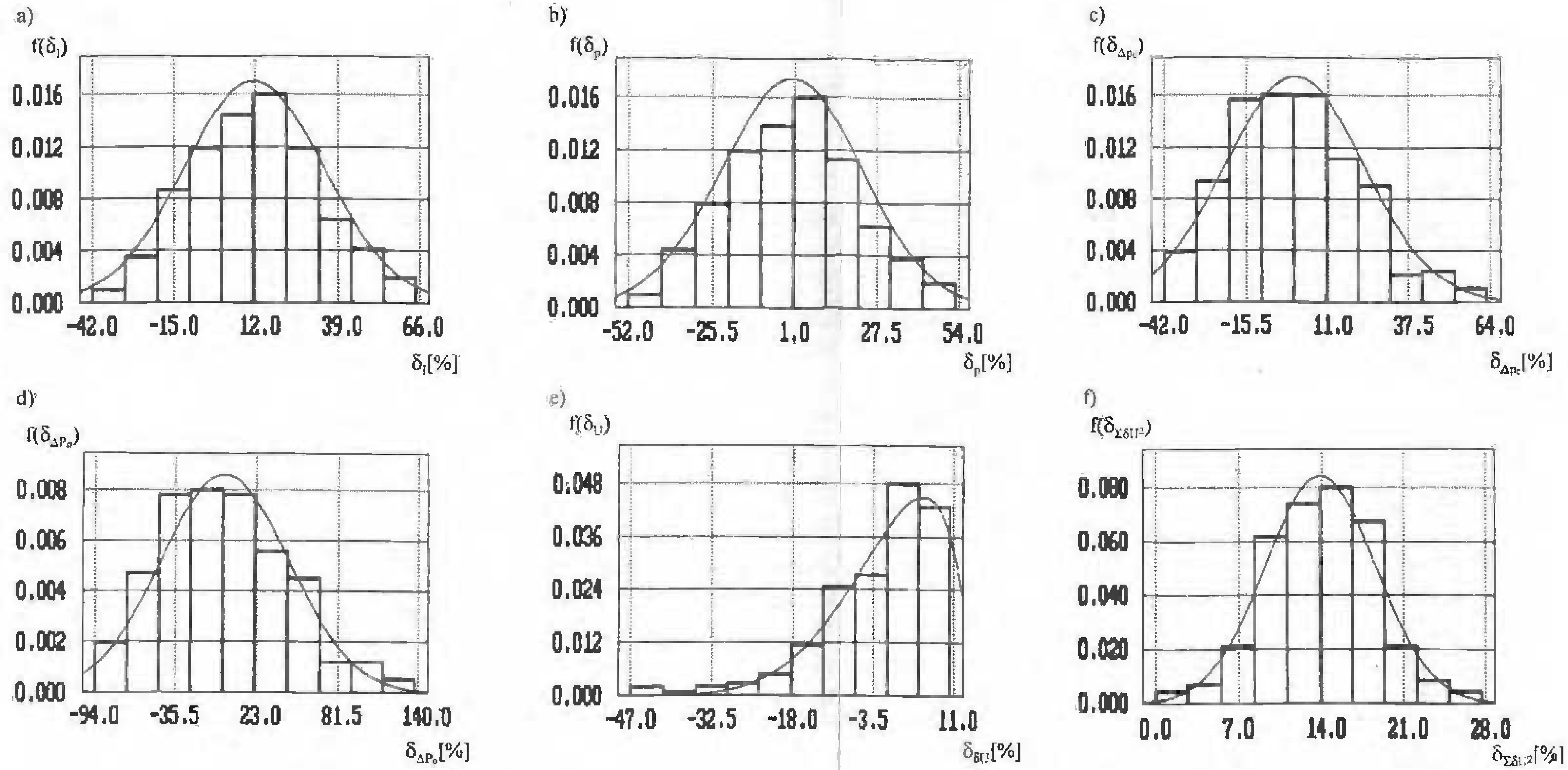


Fig. 3.3. Histograms and probability density functions for percentage errors of calculations: a) current amplitude flows in line L1, b) active power flows in line L1, c) total active power losses in the part of the system supplied from line L1, d) load active power losses in the part of the system supplied from line L1, e) voltage deviation at bus PS428, f) sum of voltage square deviation on LV receiving buses

For each realization of a load vector P^k (3.50) power flow studies were made. As a result, n_s realizations of sets of output quantities were obtained. Data acquired in this way were used to formulate and confirm probabilistic models of the output quantities.

Errors of calculations of investigated quantities on the basis of utility staff load evaluations were calculated in relation to values obtained from deterministic solutions of the power flow study for assumed average values of loads on receiving buses. In each case the histogram of observed errors was constructed (Fig. 3.2 and Fig. 3.3). On the basis of the premises of the central limit theorem [52], it was assumed that the probability distributions of the errors are normal. The statistical hypothesis was tested by the Kolmogorov-Smirnov test [8]. The test does not demand linking of data into groups and comparing the separate categories, but it compares all data in unchanged form. The Kolmogorov-Smirnov test of goodness is substantiated when the hypothetical model is assumed quite independently from data. The limitation is not practically very important if the sample size is large (of several tens order) [8].

The results of the tests of distribution goodness presented that the distribution of calculation errors of power and currents flows and the calculations of power and energy losses at the distribution system may be approximated by normal distributions. Only in the case of power and energy losses calculations for lines supplying the small number of substations ($n \leq 4$) there are no reasons, on the significance level $\alpha = 0.05$, to accept a hypothesis that the distributions are normal.

The results of the goodness tests and the parameter estimations are shown in Table 3.8.

Then - to verify the hypothesis assuming the equality of the expected values and relative error standard deviations, made during the calculation of the active power flowing into the particular MV lines and during the calculation of active power losses in distribution system fragments supplied by the particular MV lines - the statistic tests were made: the average homogeneity test and variance homogeneity test, make it possible to compare the average values and the variances for different samples [67, 115].

To compare the averages and variances the lines L1, L11, L40, L60, L77 were chosen, from which a large number of substations MV/LV is supplied and in which the relative errors of the searched quantities have the normal distributions.

In all searched cases, the goodness tests of average values of particular searched quantities gave, on the significance level $\alpha = 0.05$, the satisfactory results. With the expected value significance test, the hypothesis that all average values may be accepted as equal zero was verified.

The variance homogeneity tests gave, for the significance level $\alpha \geq 0.001$, unsatisfactory results. It means that there is no basis to accept the hypothesis that the distributions of the searched quantity errors in particular lines belong to the same population, described by one distribution $N(\mu,\delta)$.

Table 3.9

Results of the test of goodness of normal distribution fit with error distributions

Investigated object	Mean value	Standard deviation	Critical significance level
	m	σ	α
PS 149	4.9	8.6	0.006
PS 25A	2.1	22.0	0.019
PS 26	6.2	12.3	0.0004
PS 288	17.2	27.4	0.03
PS 428	-2.3	10.3	0.0008
PS 437	6.3	23.1	0.02
PS 64	-5.5	7.9	0.002
$\Sigma\delta U^2$	14.1	4.5	1

Table 3.9 shows the parameter estimations and the results of the goodness tests with the normal distribution of the distributions of relative errors made at calculations of the voltage deviations at the selected buses and the squares sum of voltage deviations at the system buses.

The results of the verification show that on the significance level $\alpha = 0.05$ there is no reason to accept a hypothesis that the voltage deviation error distributions at system buses are normal. Also the homogeneity tests of the average values and of variances gave unsatisfactory results. The calculated asymmetry coefficient values and the values of the flatness coefficients show the affiliation of the searched distributions to the class of generalized distributions beta. The distribution of the relative errors of the square sum of voltage deviations at system buses is very well approximated by the normal distribution.

The simulation experiment results were presented graphically as histograms of observed error values (Fig. 3.2 and Fig. 3.3). The same figures show the diagrams of function of theoretical distribution density selected for experimental data representation.

The statistical analysis of simulation calculation results are shown in this book and presented in publications [77, 78] let formulate the following conclusions:

1. Errors, which appear in operational practice, have the great influence for the precision of distribution system results. The simulation experiment proved that the errors of power loss estimation in distribution system about 40% and in fragments of distribution system supplied by the single lines from 70% to 100% have to be regarded (Fig. 3.2 and Fig. 3.3).
2. The obtained results proved the thesis that - regarding the fact that the information of loads are generally uncertain - most of used distribution system state optimization methods are of no practical use. Both in a case of the network optimum operation configurations and in a case of the voltage regulations, the theoretical effects of optimization (several, dozens percent [57]) are contained in uncertain area of calculations.

Now three directions which make possible to solve the optimization problem in a proper way may be suggested.

The first direction is an orientation for the load telemetry. It needs installation of a great number of measuring and transmitting instruments and development of data transmission network. In the information collecting centers the receiving devices, controlling and processing the information, and instruments for transmission of information to computational systems should be installed. The information in such a way obtained would be of a great size and could be utilized only under the condition of its earlier synthesis and statistic handling.

The second direction is inclined to the utilization of semiautomatic load and voltage local measuring instruments. The instruments take measurements automatically, and the results are stored. The measurement records are collected by the staff and delivered to the computational center. It demands less complicated technical solutions than in the first case but also needs a great expenditure of work and money.

The third direction is the utilization of incomplete primary information on loads. The most probable load values are evaluated on the basis of the accessible complementary data, connected with the load. It demands the earlier determination of the stable statistical relations between bus loads and easier available data.

In the present stage of electrical power distribution system development, the estimation of the loads at the system buses seems to be real according to the third direction with a partial utilization of two previous directions and the utilization of the statistical compensation of the telemetry deficit.

4. THE ELECTRIC POWER DISTRIBUTION SYSTEM STATE ESTIMATION

4.1. Problem formulation

The operational state of the radial power distribution system can be explicitly described - in the given time moment - by:
- the configuration of the distribution system,
- the parameters of the system elements,
- the voltage at the supplying node,
- the loads at the system buses-receipt points.

The above information makes possible the explicit formulation and solving of the problem of the power (current) flow at the system and thereby the calculation of the voltages at system buses and currents at the system branches.

The modeling of the electric power distribution system in real time [15] means the creating of the most probable data set about the configuration and the state of the system operation on the basis of the available set of observations. The problem of the real time distribution system modeling includes the defining of the distribution system configuration, the evaluation of the state estimate and the detection and the identification of the gross errors in the measurement data and signals.

This chapter describes the mathematical model and the computing techniques of the estimation of the power distribution system state vector which is the key problem of the real time power distribution system modeling.

The data concern to the distribution system configuration and to the parameters of its elements belong, in the discussed problem, to the group of the constant data. As there is not any data about the twenty-four hours' changing of the passive powers at the system nodes, the approximate calculation is taken. For each subclass of the buses-receipt points with the similar characteristics of the changes in the load time, the constant, average value of the power coefficient is assumed.

With such assumptions, the active power at the system buses-receipt points and the voltage module at the supplying point are assumed as the state variables for the electric power distribution system in the steady operation state. Having such defined state vector the calculation of all other quantities

Table 4.1.

Range of average values of aligment factors for characteristic class of customers

Class number	Type of customers	Values of aligment factors
class 1	industrial, three-shift	$0.88 \sim 1.02\ I_n$
		$0.96 \sim 1.19\ I_{pp}$
		$0.79 \sim 1.08\ I_p$
		$0.84 \sim 1.07\ I_w$
class 2	municipal-services, banks, hospitals, post-office, ambulatory, office building	$0.66 \sim 0.84\ I_n$
		$0.90 \sim 1.27\ I_{pp}$
		$1.00 \sim 1.29\ I_p$
		$0.93 \sim 1.27\ I_w$
class 3	municipal-living, blocks of flats eleven and four-storeyed, boarding-school, shops, street lighting	$0.48 \sim 0.67\ I_n$
		$0.65 \sim 0.86\ I_{pp}$
		$0.75 \sim 1.05\ I_p$
		$1.76 \sim 1.99\ I_w$
class 4	municipal-living, small one-shift industry, blocks of flats eleven and four-storeyed, hotel, shops, street lighting	$0.54 \sim 0.80\ I_n$
		$0.74 \sim 1.17\ I_{pp}$
		$0.91 \sim 1.25\ I_p$
		$1.24 \sim 1.87\ I_w$
class 5	living, blocks of flats four-storeyed	$0.34 \sim 0.41\ I_n$
		$0.69 \sim 0.83\ I_{pp}$
		$1.19 \sim 1.27\ I_p$
		$1.82 \sim 2.03\ I_w$
class 6	broadcast transmitter	$0.63 \sim 0.71\ I_n$
		$1.14 \sim 1.19\ I_{pp}$
		$1.11 \sim 1.18\ I_p$
		$1.11 \sim 1.13\ I_w$

which are interesting from the distribution system operation control point of view can be performed.

The set of the measurements made at the power distribution systems currently consists of:

- the voltage modules at the MV and HV busbars of the HV/MV transformer which supplies the system,
- the flow of the active and passive power in the HV/MV transformer which supplies the system,
- the modules of the currents at the MV feeders in HV/MV substations.

The observation set is completed by the evaluations of the active powers received at the nodes-receipt points (usually the LV busbars of the MV/LV transformers).

4.2. The static state estimation

The static state estimation assumes that during the data acquisition and processing the changes in the distribution system do not appear, so the data is concerned with the same steady state.

In this book the two-stages algorithm of the load estimation at the system buses is suggested. In the first part of the algorithm the initial load diagrams at the system buses are created on the basis of available information. In the second part the initial load diagrams are corrected on the basis of the data given by telemetry. In such approach the obtained values of the state variables are fitted to the existing set of observations in the best possible manner.

4.2.1. The classification of the customers into the characteristic groups

The graphic illustration of the load curve during the 24-hour is the 24-hour calendar load diagram $P_{dt} = f(t_{dk})$ (Fig. 4.1). There are the following characteristic load values at the diagram:

P_{ds} - the peak load
P_{dsr} - the average load,
P_{do} - the base load,
P_{di} - the installed capacity of the electric energy receivers.

To differ the received electrical energy according to the level of the demand and to the moment of energy consumption, the division of the 24-hour diagram into the horizontal layer and the vertical columns is introduced [70] (Fig. 4.1).

The daily load profile may be characterized by three groups of coefficients:
- the degree of alignment (l_d),
- the degree of exploitation (n_d),
- the degree of load (m_d),

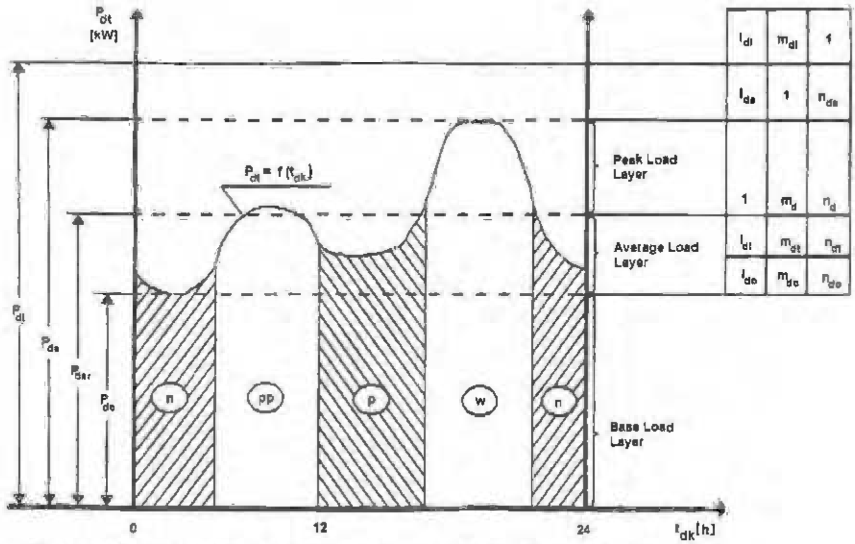


Fig. 4.1. The 24-hour load profile: the division into the layers and columns.
Designations: n - the night load, pp - the morning load, p - the afternoon load, w - the evening load.

The 24-hour load profile of the different customer groups show the characteristic variation. Considering a big number of the customers in the power distribution system and the lack of the measuring and recording stationary devices at the system buses, the analysis of variation of the 24-hour load at the individual buses is not possible. The buses-receive points are grouped into classes according to the load profile. To each class the typical 24-hour profile characteristic for the given class of receivers is assigned. On the basis of the analysis of the shapes of the 24-hour profiles of the different receivers during work days, the four characteristic intervals of the 24-hours are distinguished:

- night	22.00 - 6.00 h,
- morning	6.00 - 13.00 h,
- afternoon	13.00 - 6.00 h,
- evening	16.00 - 22.00 h,

The classification of the receivers according to the 24-hour load profiles is made on the basis of the average alignment degree for each column. To avoid the influence of the instantaneous values on the changes of power consumed by the receiver, the average load of the 24-hours should be taken as the reference quantity. The average alignment degree at the column is:

$$l_j = \frac{P_{srj}}{P_{dsr}}, \tag{4.1}$$

where:

j - the column index (n, pp, p, w),

P_{srj} – the average power at the column j,

P_{dsr} – the average value of the load of 24-hour load profile.

The load diagrams are regarded as similar ones if their average alignment degrees in the characteristic intervals of 24-hours have the similar values.

To distinguish the particular classes, the four-dimensional space with l_n, l_{pp}, l_p, l_w coordinates is constructed. Then, it is divided into hypercubicoides with Δl_n, Δl_{pp}, Δl_p, Δl_w sides. Each 24-hour load profile is represented within the space by the point with coordinates which correspond to the alignment degrees calculated for the profile. The exemplary figure for the two-dimensional space is presented in Fig. 4.2.

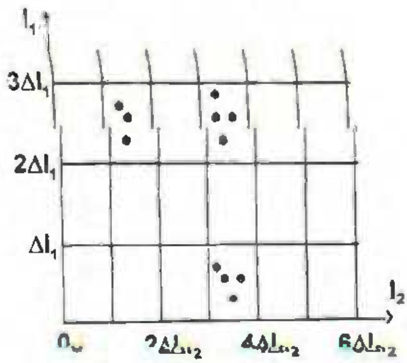


Fig. 4.2. The example of grouping of the 24-hour load profiles on the plane l_1, l_2

The tendency of grouping the points (which correspond the 24-hour load profiles) at several areas is observed.

The receivers for which the points corresponding their load profiles are grouped at the same hypercubicoid are regarded as the same customer class.

For small length of the hypercubicoid sides ($\Delta l_j \leq 0.2$) a great number of classes which consist only of single profiles was obtained. The satisfactory results of grouping (several classes) are obtained for the values:

$$\Delta l_n = 0{,}3;\ \Delta l_{pp} = 0{,}3;\ \Delta l_p = 0{,}3;\ \Delta l_w = 0{,}4.$$

Six characteristic load classes [78] were obtained as a result of the classification of the real measurements of the 24-hour load profiles at the system buses-receipt points. The results of grouping of the measurement data are presented at the Tab. 4.1.

The method of the construction of the typical 24-hour load profile for the particular class consists of the following steps:

1. The calculations of the average 24-hour load profile for the given class

The average 24-hour load profile for the given class is calculated on the basis of the following dependence:

$$P_j(k) = \frac{1}{L} \sum_{l \in \alpha_j} P_l(k), \qquad (4.2).$$

where:

j - the index of the selected customer class,
k - the index of the quarter of an hour of the 24-hours,
L - the number of the 24-hour load profiles classified to the class k,
α_j - the set of the indexes of the 24-hour load profile classified to the class j.

2. The elimination of the instantaneous random load variations

The instantaneous random load variations of the obtained average profiles are eliminated by one of the operations which is equivalent to the low-pass filtration. To smooth the load curves the method based on the discrete Fourier transform was taken [84, 99].
There are the following stages of smoothing the data:

a) To calculate the discrete Fourier transform for the series (4.2)

$$P_j(n) = \sum_{k=0}^{N-1} P_j(k) \exp(-j2\pi k n/N), \qquad (4.3)$$

$$n = 0, 1, 2 \ldots N-1,$$

where N is the sample length.

Table 4.1.

Range of average values of alignment degrees for characteristic classes of customers

Class number	Type of customers	Values of alignment factors
class 1	industrial, three shift	0.88~1.02 I_n
		0.96~1.19 I_{pp}
		0.79~1.08 I_p
		0.84~1.07 I_w
class 2	municipal-services, banks, hospitals, post-office, ambulatory, office buildings	0.66~0.84 I_n
		0.90~1.27 I_{pp}
		1.00~1.29 I_p
		0.93~1.27 I_w
class 3	municipal-living, blocks of flats eleven and four-storeyed, boarding-school, shops, street lighting	0.48~0.67 I_n
		0.65~0.86 I_{pp}
		0.75~1.05I_p
		1.76~1.99 I_w
class 4	municipal-living, small one-shift industry, blocks of flats eleven and four-storeyed, hotel, shops, street lighting	0.54~0.80 I_n
		0.74~1.17 I_{pp}
		0.91~1.25 I_p
		1.24~1.87 I_w
class 5	living, blocks of flats four-storeyed	0.34~0.41 I_n
		0.69~0.83 I_{pp}
		1.19~1.27 I_p
		1.82~2.03 I_w
class 6	broadcast transmitter	0.63~0.71 I_n
		1,14~1.19 I_{pp}
		1.11~1.18 I_p
		1.11~1.13 I_w

b) To use the spectral window to the Fourier transform (4.3)

$$\hat{P}_j(n) = \omega(n)\,P_j(n), \qquad (4.4)$$

where $\omega(n)$ is the viewing function describing the spectral window

c) To calculate the inverse discrete Fourier transform

$$\hat{\hat{P}}_j(k) = \frac{1}{N}\sum_{n=0}^{N-1}\hat{P}_j(n)\exp(j2\pi k n/N), \qquad (4.5)$$

$$k = 0, 1, 2, \ldots, N-1$$

3. The normalization of the values $\hat{P}_j(k)$ in relation to the maximum values

$$\overline{P}_j(k) = \frac{\hat{P}_j(k)}{P_{js}}, \qquad (4.6)$$

where

$$P_{js} = \max_k\{\hat{P}_j(k)\}. \qquad (4.7)$$

The $\overline{P}_j(k)$ sequence, calculated according to the formula (4.6), corresponds to the typical load profile of the given class, normalized in relation to the peak power.

For the practical calculations the rectangle window with the following viewing function is used:

$$\omega(n) = \begin{cases} 0 & 0 \le n \le m \\ 1 & \text{for other } n \end{cases}, \qquad (4.8)$$

where m is the width of the window in the frequency domain.

The acceptance of the above window means the omitting of the process components with the frequencies

$$f_s > \frac{m}{N h}, \qquad (4.9)$$

where h is the sampling interval.

On the basis of the practical calculations for the N=96 and h=15 minutes was stated that the most satisfactory results are obtained while the width of the window is m=8 which corresponds to the omission of the run components of the frequencies bigger than $f_s \approx 9.26 \times 10^{-5}$ Hz (T_s = 3 h).

Fig. 4.3 and Fig. 4.4 present the typical 24-hour load profiles for two selected customer classes, calculated according to the procedure described above.

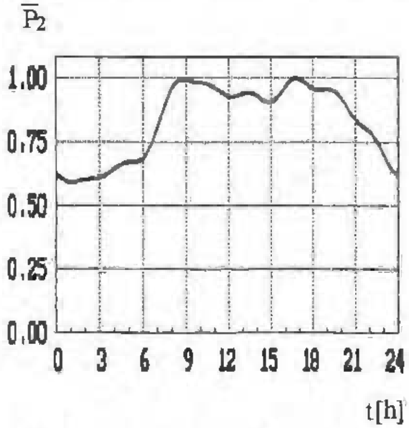


Fig. 4.3. Typical 24-hour load profile for class 2 customers (municipal-commercial)

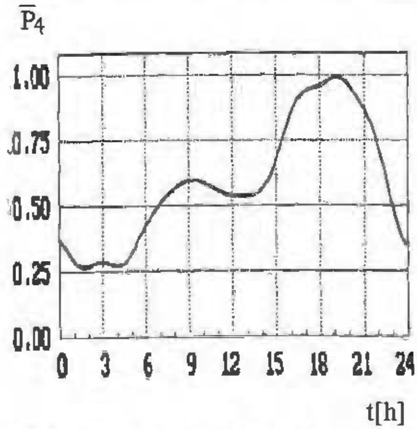


Fig. 4.4. Typical 24-hour load profile for class 4 customers (municipal-living)

As a measure of accuracy of approximation of real curves by typical profiles the relative mean-square deviation was taken

$$\eta = \sqrt{\frac{1}{96}\sum_{k=0}^{95}[\overline{P}_r(k) - \overline{P}_j(k)]^2}, \qquad (4.10)$$

where:

$\overline{P}_r(k)$ – the real load run normalized in relation to the peak load,

$\overline{P}_j(k)$ – the typical load profile of j class of customer.

The results of the calculation of the relative mean-square deviation for the selected 24-hour load profiles of the particular customer classes show that η value usually is of several - dozen percent and it does not generally exceed the value of 20 %. The bigger error values were stated in cases when the receivers of large, short-time power demand have considerable participation in the substation load.

The typical load profiles are utilized for the estimation of real runs. According to the selected additional information, the following procedures can be used [69]:

1. The peak load of the receiver is known

$$P_r(k) = P_s \overline{P}_j(k), \tag{4.11}$$

where P_s is the receiver peak load.

2. The power demanded by the receiver at the selected hour is known

$$P_r(k) = \frac{P_r(t)}{\overline{P}_j(t)} \overline{P}_j(k), \tag{4.12}$$

where:

$\overline{P}_j(t)$ – the power demanded by the receiver at the hour t,

$P_r(t)$ – the relative load value at the hour t according to the typical load profile.

3. The 24-hour electric energy consumption is known

$$P_r(k) = \frac{W_d}{N} \overline{P}_j(k), \tag{4.13}$$

where:

W_d – the 24-hour electric energy consumption,

N - the number of time-intervals in the 24-hour period.

4. The monthly consumption of electric energy is known

$$P_r(k) = \frac{W_m}{N D_{sr}} \overline{P}_j(k), \tag{4.14}$$

where:

D_{sr} – the monthly consumption of electric energy,

W_m – the equivalent number of work days in a month.

It is the intended selected number of working days during which the electric energy equals the real consumed energy which would be consumed.

5. The average level of transformer load for the specified class of customer is known

$$P_r(k) = k_j \overline{P}_j(k) S_n , \tag{4.15}$$

where:

k_j – the average level of transformer load for the specified class of customer,

S_n – the rated power of a transformer installed at the receiving substation.

4.2.2. The state vector estimation

The mathematical model of the state vector estimation problem for the nonlinear stationary system without dynamics has a following form [96]:

$$\mathbf{z} = \mathbf{h}(\mathbf{x}) + \mathbf{v} , \tag{4.16}$$

where:

- $\mathbf{z}$ - m-dimensional observation vector,
- $\mathbf{x}$ - n-dimensional state vector,
- $\mathbf{h}(\mathbf{x})$ - m-dimensional vector - prediction function relating, on the basis of Ohm and Kirchhoff laws, the observation vector to the state vector,
- $\mathbf{v}$ - m-dimensional vector of the observation errors.

For radially operated power distribution systems of medium voltage the magnitude of supplying bus voltage and active power load at the system buses-receipt points can be chosen as state variables in the steady-state system operation. So, the vector of state variables is defined by the following equation:

$$\mathbf{x}^T = [V_0, P_1, P_2, \ldots, P_i, \ldots, P_n], \tag{4.17}$$

where:

V_0 - the voltage module at the supplying bus,

P_i – the active power received at bus i,

n - the number of buses-receipt points.

The observation vector looks like:

$$
\begin{aligned}
\mathbf{z}^T = [&V_0, V_1, \ldots, V_u, \ldots, V_U, I_T, I_1, \ldots, I_l, \ldots, I_L, \\
&p_T, p_1, \ldots, p_k, \ldots, p_K, q_T, q_1, \ldots, q_r, \ldots, q_R, \\
&P_1, \ldots, P_w, \ldots, P_W, Q_1, \ldots, Q_s, \ldots, Q_S]
\end{aligned}
\tag{4.18}
$$

In the above formula:

V_0 – the measurement of the voltage module at the supplying bus,

V_u – the measurement of the voltage module at the system buses

I_T – the measurement of the current magnitude in the supplying transformer,

I_l – the measurement of the current magnitude in the branch l,

p_T – the measurement of the active power flow in the supplying transformer,

p_k – the measurement of the active power flow in the branch k,

q_T – the measurement of the passive power flow in the supplying transformer,

q_r – the measurement of the passive power flow in the branch r,

P_w – the measurement of the active power received at the bus w,

Q_s – the measurement of the passive power received at the bus s,

U - the number of buses with the measurement of voltage magnitude,
L - the number of branches with the measurement of current magnitude,
K - the number of branches with the measurement of the active power flow,
R - the number of branches with the measurement of passive power flow,
W - the number of buses with the measurement of active power,
S - the number of buses with the measurement of passive power.

The vector of observation errors, corresponding to the observation vector, has a following form:

$$
\mathbf{v}^T = [v_1, \ldots, v_j, \ldots, v_m], \tag{4.19}
$$

where v_j is the error of observation j.

Assuming the mathematical model of the system as it was described in the section 4.2 and taking into account the accepted simplifying assumptions it is possible to derive the dependencies between the observed quantities and the state variables. For the particular observations those dependencies can be written as follows.

The voltage magnitude at the supplying bus

$$V_Q = V_0 \tag{4.20}$$

The voltage magnitude at the receiving buses

$$V_u = V_0 - V_0^{-1} \mathbf{b}_u (\mathbf{R}_L - \mathbf{X}_L \,\mathbf{tg}\varphi)\mathbf{b}^T \mathbf{P}, \tag{4.21}$$

where:

b - branch-path incidence matrix, where path is oriented from the bus u to the reference node (the supplying bus). The elements of the matrix **b** are as follows:

$$b_{u,j} = \begin{cases} 1, & \text{if the branch j is in the path u,} \\ 0, & \text{if the branch j is not in the path u,} \end{cases} \tag{4.22}$$

$\mathbf{b}_u$ - the row vector of matrix **b** corresponding to the path d_u from bus u to the reference node (the supplying bus),

P - the vector of active powers received at the system buses,

tgφ - the diagonal matrix of ratios of passive power to active power received at the system nodes,

$\mathbf{R}_L$ - the diagonal matrix of branch resistances,

$\mathbf{X}_L$ - the diagonal matrix of branch reactances.

The magnitude of the current flow in the supplying transformer

$$I_T = (\sqrt{3}\, V_0)^{-1} \sqrt{\mathbf{P}^T (\mathbf{1}_{nxn} + \mathbf{tg}\varphi\, \mathbf{1}_{nxn}\, \mathbf{tg}\varphi)\mathbf{P}}\,, \tag{4.23}$$

where $\mathbf{1}_{n \times n}$ is the square matrix all elements of which equal one.

The magnitude of the current flow in the branch l

$$I_l = (\sqrt{3}\, V_0)^{-1} \sqrt{\mathbf{P}^T (\mathbf{b} \mathbf{1}_{l,l} \mathbf{b}^T + \mathbf{tg}\varphi \mathbf{b} \mathbf{1}_{l,l} \mathbf{b}^T \mathbf{tg}\varphi)\mathbf{P}}\,, \tag{4.24}$$

where $\mathbf{1}_{l,l}$ is the square matrix the element $\delta_{l,l}$ of which equals one and the other elements are zero.

The active power flow in the supplying transformer

$$p_T = \mathbf{1}^T_{nx1}\,\mathbf{P}\,, \tag{4.25}$$

where $\mathbf{1}_{nx1}$ is the vector all elements of which equal one.

The active power flow in branch k

$$p_k = \mathbf{1}^T_k\,\mathbf{b}^T\mathbf{P}\,, \tag{4.26}$$

where $\mathbf{1}_{nx1}$ is the vector the element δ_k of which is equal one and the other elements are zero.

The passive power flow in the supplying transformer

$$q_T = \mathbf{1}^T_{nx1}\,\mathbf{tg\varphi P}\,. \tag{4.27}$$

The passive power flow in the branch r

$$q_r = \mathbf{1}^T_r\,\mathbf{b}^T\,\mathbf{tg\varphi P}\,. \tag{4.28}$$

The active power received at the bus w

$$P_w = P_w\,. \tag{4.29}$$

The passive power received at the bus s

$$Q_s = \mathrm{tg}\varphi_s\,P_s\,. \tag{4.30}$$

The solution method of the state vector estimation problem is based on the linearization of nonlinear prediction function $\mathbf{h}(\mathrm{x})$ (4.16). The linear system model is obtained by the expansion of the prediction function in the Taylor series in the neighbourhood of $\mathbf{x}_0$ and by the rejecting of terms of the series higher than the second order.

Therefore, the expression describing the linear model of the system is [96]

$$\mathbf{z} - \mathbf{h}(\mathbf{x}_0) = \mathbf{H}(\mathbf{x} - \mathbf{x}_0) + \mathbf{v} , \tag{4.31}$$

where:

$\mathbf{H}$ - m×n-dimensional Jacobian sensitivity matrix,

$\mathbf{x}_0$ – the initial value of the state vector estimation.

Elements of the matrix $\mathbf{H}$ are calculated from the equation

$$\mathbf{H} = \left.\frac{\partial \mathbf{h}}{\partial \mathbf{x}}\right|_{\mathbf{x}=\mathbf{x}_0} \tag{4.32}$$

The solution of thus formulated problem consists in the determination of such an estimator (observation function) which secures the best, in the certain specified meaning, estimate of the state vector.

In this case the estimator will be the linear matrix operator

$$\hat{\mathbf{x}} = \mathbf{W}\,\mathbf{z} , \tag{4.33}$$

where:

$\hat{\mathbf{x}}$ - the estimate of x value,

$\mathbf{W}$ - the estimator.

In this book the approach based on the idea of the weighted least-squares estimator is proposed [96]. Such an estimation of the vector x is searched, which minimizes the expression

$$I(\mathbf{x}) = \left\| [\mathbf{z} - \mathbf{h}(\mathbf{x}_0)] - \mathbf{H}(\mathbf{x} - \mathbf{x}_0) \right\|^2_{\mathbf{R}^{-1}} , \tag{4.34}$$

where $\mathbf{R}^{-1}$ is the positively defined weight matrix.

The condition (4.34) can be written as follows

$$I(\hat{\mathbf{x}}) = [\mathbf{z} - \mathbf{h}(\mathbf{x}_0) - \mathbf{H}(\hat{\mathbf{x}} - \mathbf{x}_0)]^T \mathbf{R}^{-1} [\mathbf{z} - \mathbf{h}(\mathbf{x}_0) - \mathbf{H}(\hat{\mathbf{x}} - \mathbf{x}_0)] = \min \tag{4.35}$$

The solution of the equation (4.35) is

$$\hat{\mathbf{x}} = \mathbf{x}_0 + [\mathbf{H}^T(\mathbf{x}_0)\mathbf{R}^{-1}\mathbf{H}(\mathbf{x}_0)]^{-1}\mathbf{H}^T(\mathbf{x}_0)\mathbf{R}^{-1}[\mathbf{z} - \mathbf{h}(\mathbf{x}_0)] . \tag{4.36}$$

Considering the omission of the higher order terms in the expansion of the function h(x) and the acceptance of $\mathbf{x}_0$ as an initial estimation, the expression (4.36) should be considered as iterative [49]

$$\hat{\mathbf{x}}_{l+1} = \hat{\mathbf{x}}_l + [\mathbf{H}^T(\hat{\mathbf{x}}_l)\mathbf{R}_l^{-1}\mathbf{H}(\hat{\mathbf{x}}_l)]^{-1}\mathbf{H}^T(\mathbf{x}_l)\mathbf{R}_l^{-1}[\mathbf{z} - \mathbf{h}(\hat{\mathbf{x}}_l)], \quad (4.37)$$

calculated till the moment when

$$\left|\hat{\mathbf{x}}_l - \hat{\mathbf{x}}_{l-1}\right| < \varepsilon, \quad (4.38)$$

where:

l - iteration number,

ε – accepted in advance precision of the calculations.

The matrix $[\mathbf{H}^T\mathbf{R}^{-1}\mathbf{H}]^{-1}$ exists when the dimension of the vector $\mathbf{z}$ is not smaller than the dimension of the vector $\mathbf{x}$, i.e. $m \geq n$.

The estimation methods based on the minimization of the weighted least-squares sum do not need the introduction of any uncertainty models for vectors $\mathbf{x}$ and $\mathbf{v}$. The weight matrix $\mathbf{R}^{-1}$ is selected on the basis of engineering discernment [96].

The elements of the matrix **H** are calculated on the basis of the dependence (4.32) from the following equations:

$$\frac{\partial V_0}{\partial V_0} = 1, \quad (4.39)$$

$$\frac{\partial V_0}{\partial P_i} = 0, \quad (4.40)$$

$$\frac{\partial V_u}{\partial V_0} = 1 + V_0^{-2}\,\mathbf{b}_u(\mathbf{R}_L - \mathbf{X}_L\,\mathbf{tg}\varphi)\mathbf{b}^T\mathbf{P}, \quad (4.41)$$

$$\frac{\partial V_u}{\partial P_i} = -V_0^{-1}\,\mathbf{b}_u(\mathbf{R}_L - \mathbf{X}_L\,\mathbf{tg}\varphi)\mathbf{b}^T\mathbf{1}_i, \quad (4.42)$$

$$\frac{\partial I_T}{\partial V_0} = -\frac{1}{\sqrt{3}} V_0^{-2} \sqrt{\mathbf{P}^T(\mathbf{1}_{nxm} + \mathbf{tg\varphi} \mathbf{1}_{nxm} \mathbf{tg\varphi})\mathbf{P}}\,, \tag{4.43}$$

$$\frac{\partial I_T}{\partial P_i} = (\sqrt{3}\, V_0^{-2})^{-1} [\mathbf{P}^T(\mathbf{1}_{nxm} + \mathbf{tg\varphi} \mathbf{1}_{nxm} \mathbf{tg\varphi})\mathbf{P}]^{-1/2} \times$$
$$\times \mathbf{P}^T(\mathbf{1}_{nxm} + \mathbf{tg\varphi} \mathbf{1}_{nxm} + \mathbf{tg\varphi} \mathbf{1}_{nxm} \mathbf{tg\varphi}) \mathbf{1}_i\,, \tag{4.44}$$

$$\frac{\partial I_l}{\partial V_0} = -\frac{1}{\sqrt{3}} V_0^{-2} \sqrt{\mathbf{P}^T(\mathbf{b}\, \mathbf{1}_{l,l} \mathbf{b}^T + \mathbf{tg\varphi}\, \mathbf{b}\, \mathbf{1}_{l,l} \mathbf{b}^T \mathbf{tg\varphi})\mathbf{P}}\,, \tag{4.45}$$

$$\frac{\partial I_l}{\partial P_i} = (\sqrt{3}\, V_0^{-2})^{-1} [\mathbf{P}^T(\mathbf{b}\, \mathbf{1}_{l,l} \mathbf{b}^T + \mathbf{tg\varphi}\, \mathbf{b}\, \mathbf{1}_{l,l} \mathbf{b}^T \mathbf{tg\varphi})\mathbf{P}]^{-1/2} \times$$
$$\times \mathbf{P}^T(\mathbf{b}\, \mathbf{1}_{l,l} \mathbf{b}^T + \mathbf{tg\varphi}\, \mathbf{b}\, \mathbf{1}_{l,l} \mathbf{b}^T \mathbf{tg\varphi}) \mathbf{1}_i\,, \tag{4.46}$$

$$\frac{\partial p_T}{\partial V_0} = 0\,, \tag{4.47}$$

$$\frac{\partial p_T}{\partial P_i} = 1\,, \tag{4.48}$$

$$\frac{\partial p_k}{\partial V_0} = 0\,, \tag{4.49}$$

$$\frac{\partial p_k}{\partial P_i} = \mathbf{1}_k^T \mathbf{b}^T \mathbf{1}_i\,, \tag{4.50}$$

$$\frac{\partial q_T}{\partial V_0} = 0, \tag{4.51}$$

$$\frac{\partial q_T}{\partial P_i} = \mathbf{1}_{nx1}^T \mathbf{tg\varphi 1}_i, \tag{4.52}$$

$$\frac{\partial I_T}{\partial V_0} = -\frac{1}{\sqrt{3}} V_0^{-2} \sqrt{\mathbf{P}^T(\mathbf{1}_{nxm} + \mathbf{tg\varphi}\,\mathbf{1}_{nxm}\,\mathbf{tg\varphi})\mathbf{P}}, \tag{4.53}$$

$$\frac{\partial q_r}{\partial P_i} = \mathbf{1}_r^T \mathbf{b}^T \mathbf{tg\varphi}\,\mathbf{1}_i, \tag{4.54}$$

$$\frac{\partial P_w}{\partial V_0} = 0, \tag{4.55}$$

$$\frac{\partial P_w}{\partial P_i} = \mathbf{1}_w^T\,\mathbf{1}_i, \tag{4.56}$$

$$\frac{\partial Q_s}{\partial V_0} = 0, \tag{4.57}$$

$$\frac{\partial Q_s}{\partial P_i} = \mathbf{1}_s^T\,\mathbf{tg\varphi 1}_i \tag{4.58}$$

The elements r_i^{-1} of the matrix $\mathbf{R}^{-1}$ correspond to the relative weights (importance) which are attributed to each observation z_k under determination of the vector $\hat{x}$. These weights are determined by the uncertainty level of the particular observations.

Matrix $\mathbf{R}^{-1}$ looks as follows:

$$\mathbf{R}^{-1} = \begin{bmatrix} r_1^{-1} & & & & & & 0 \\ & r_2^{-1} & & & & & \\ & & \ddots & & & & \\ & & & r_j^{-1} & & & \\ & & & & \ddots & & \\ & 0 & & & & & r_m^{-1} \end{bmatrix} \tag{4.59}$$

Powers received at the system buses are limited regarding the physical properties and technical conditions.

$$0 \le P_i \le K_i S_{n,i} \cos\varphi_i , \tag{4.60}$$

where

$S_{n,i}$ - the rated power of the transformer installed at the bus i,

K_i - the coefficient of the admissible overload capacity of the transformer.

Defined in such a way the variability area of the vector **x**, is divided into three zones[34]: the permitted zone and two limiting zones, each of the width

$$\alpha_i = \beta K_i S_{n,i} \cos\varphi_i , \tag{4.61}$$

where β is the coefficient of the width of the limiting zone.

The condition (4.60) is fulfilled by the proper modification of weight coefficients in the matrix $\mathbf{R}^{-1}$. These coefficients are modified in the calculation process as follows:

If the $\mathbf{x}_l$ point is situated in the permitted zone, the matrix $\mathbf{R}_l^{-1}$ remains in its previous form

$$\mathbf{R}_l^{-1} = \mathbf{R}_{l-1}^{-1} \qquad (4.62)$$

If any of the components $x_{l,i}$ of the vector $\mathbf{x}_l$ exceeds of the permitted zone then the value of the element $x_{l,i}$ is

$$x_{l,i} = \begin{cases} \alpha_i, & \text{if the left bound is exceeded,} \\ (1-\beta)K_i S_{n,i} \cos\varphi_i, & \text{if the right bound is exceeded,} \end{cases} \qquad (4.63)$$

and the value of the corresponding weight coefficient is modified as follows

$$r_{l,i}^{-1} = \lambda_i \, r_{l-1,i}^{-1}, \qquad (4.64)$$

where λ_i is the coefficient which increases the value of the proper weight.

Having such a representation of matrices and vectors it is possible to determine the estimator $\hat{\mathbf{x}}$ of the state vector $\mathbf{x}$ numerically. In this approach the obtained values of the state variables are optimally fitted (in a sense of the weighted least squares method) to the existing observation set. The method is flexible. It allows to use different sets and different number of observations and it makes it possible to give proper consideration to the uncertainty level of the particular observations. When there is not a sufficient number of measurements ($m \geq n$), the missing measurements can be replaced by pseudo-measurements, i.e. by their forecast.

The following algorithm is used to calculate the vector $\mathbf{x}$.

1. Calculate

$$\mathbf{A}_l = \mathbf{H}^T(\hat{\mathbf{x}}_l)\mathbf{R}_l^{-1}\mathbf{H}(\mathbf{x}_l). \qquad (4.65)$$

2. Calculate

$$\mathbf{b}_l = \mathbf{H}^T(\hat{\mathbf{x}}_l)\mathbf{R}_l^{-1}[\mathbf{z} - \mathbf{h}(\hat{\mathbf{x}}_l)], \qquad (4.66)$$

3. Solve, considering $\Delta\mathbf{x}$, the linear equation

$$\mathbf{A}_l \Delta\mathbf{x}_l = \mathbf{b}_l , \tag{4.67}$$

where

$$\Delta\mathbf{x}_l = \hat{\mathbf{x}}_{l+1} - \hat{\mathbf{x}}_l . \tag{4.68}$$

4. Calculate

$$\hat{\mathbf{x}}_{l+1} = \hat{\mathbf{x}}_l + \Delta\mathbf{x}_l . \tag{4.69}$$

To solve the equation (4.67), the method of square-root factorization, which is characterized by high numerical stability [36,106].

4.3 The dynamic state estimation

4,3.1. The stochastic properties of the process of load variation

The power demand at each bus of the distribution system is a random time function. The stochastic process is a convenient mathematical model to describe the probabilistic structure of the load process [6, 7, 99].

Seven consecutive realizations of the 24-hour load demand processes in the selected feeder of the municipal distribution system of medium voltage are shown in Fig. 4.5 [119].

If signals are numerically processed, there is a need for their digitization. The signal sampling is usually done with the same time interval. The problem is to determine the proper sampling interval Δt, in such a way to allow explicit reproduction of the sampled signal. To determine the frequency component (the spectral line) of the original signal we must have at least two samples in the period. So, the highest frequency signal component which may be determined under sampling with the interval Δt is

$$f_g = \frac{1}{2\,\Delta t} , \tag{4.70}$$

The components of the original signal with the frequency higher than value f_g will be moved to the range $0 \div f_g$ and mixed (spliced) with the components of that lower range [6].

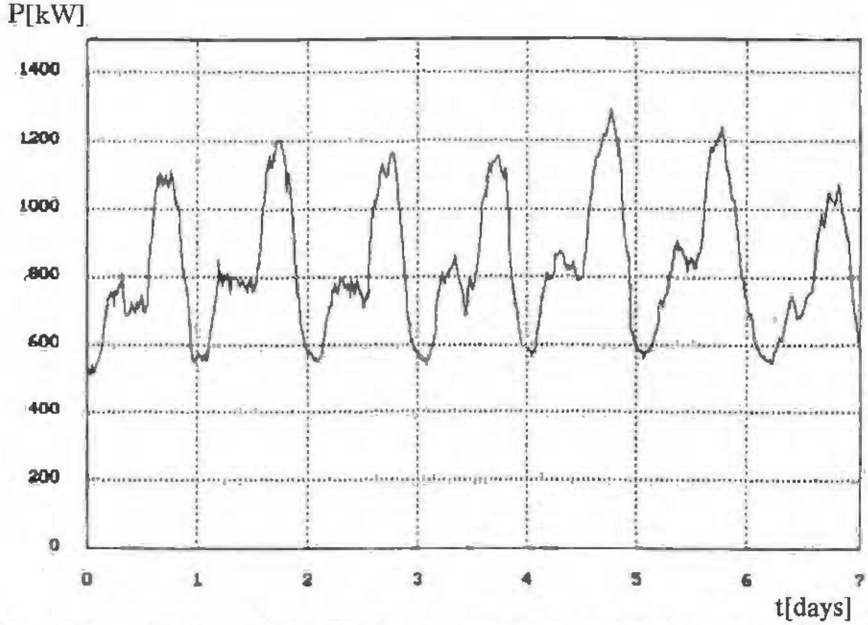


Fig. 4.5 The realizations of the load demand process in the selected feeder of the municipal power distribution system

Considering the aim of processing that is the analysis of the electric power distribution system operation in the normal steady-states, $\Delta t = 15$ minutes is accepted as the value of the sampling interval.

The sequence of the sample values of consecutive N observations

$$\{P(k)\} = (P(0), P(1), \ldots, P(k), \ldots, P(k-1)), \tag{4.71}$$

is understood as the sample taken from the infinite number of samples, which may be generated by the given stochastic process $\{P(k)\}$ (Fig. 4.6). The basic aim of the statistical considerations is to conclude the properties of the population from the properties of the samples.

The realizations of the power demand processes at the electric power distribution system buses in the chronologically arranged 24-hour, weekly and annual time intervals are characterized by the typical, cyclic repeated form. It is the symptom of nonstationarity of these processes.

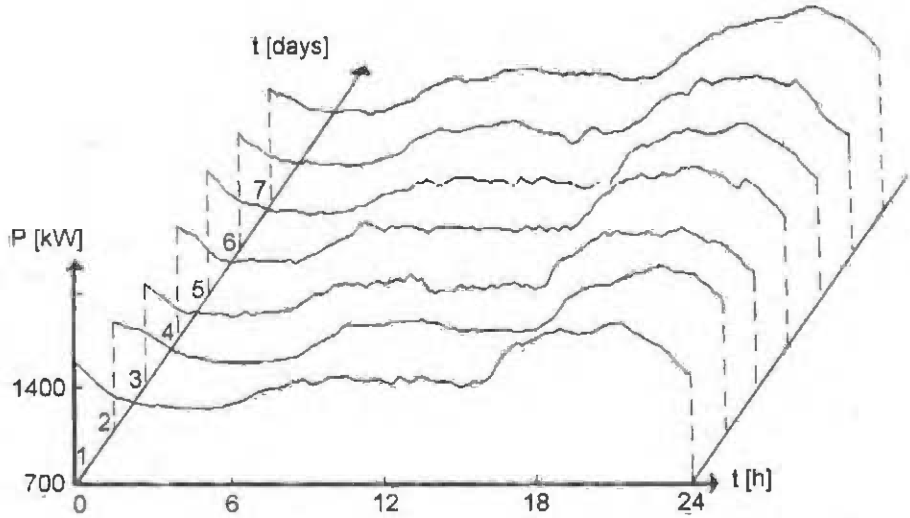


Fig. 4.6. The set ofthe realizations of the 24-hour load demand which forms the random process

The suitable tool for detection of the determined periodic components masked by the random noise is the autocorrelation function, which is the measure of the dependence between the values of stochastic process observed at different moments of time [16].

The autocorrelation function of the stochastic discrete process can be estimated from the following equation

$$R_p(k,m) = \frac{1}{L}\sum_{i=1}^{L} P_i(k)P_i(k+m), \qquad (4.72)$$

$$m = 0, 1, \ldots, N-1$$

where:

m - the shift in time between two points of the process realization,

L - the number of the process realizations.

Fig. 4.7 shows the plot of the autocorrelation function of the load process of the selected feeder calculated on the basis of the feeder load measurements for three weeks.

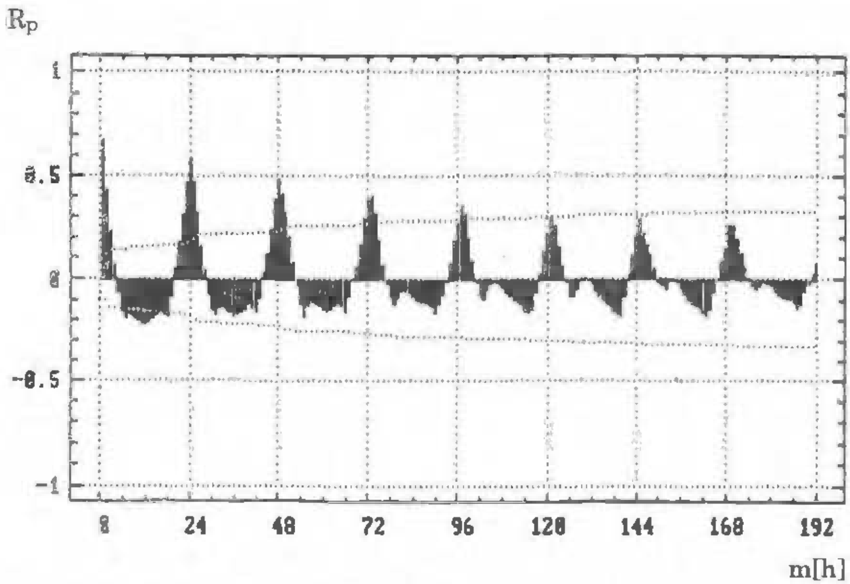


Fig. 4.7. The autocorrelation function of the load process for selected feeder of medium voltage

The spectral structure of the random process is well described by the power spectral density which may be defined as the Fourier transform of the autocorrelation function (4.72). The spectral density estimator obtained from the finite fragment of the process realization is called periodogram. Its definition is as follows [87]:

$$G_p(k,n) = \sum_{m=0}^{N-1} R_p(k,m)\exp(-j2\pi nm/N). \qquad (4.73)$$

The values $G_p(k, n)$ are the measure of the power of n-harmonic signal component.

Fig. 4.8 shows the periodogram of the load process for selected feeder of medium voltage distribution system.

The periodical form of the autocorrelograms (Fig. 4.7) shows that the periodicity is the main attribute which characterizes the load processes at the buses of a distribution system. The analysis of the load process periodograms (Fig. 4.8) shows that within the considered time horizon (several weeks) the 24-hour periodicity is the best exposed.

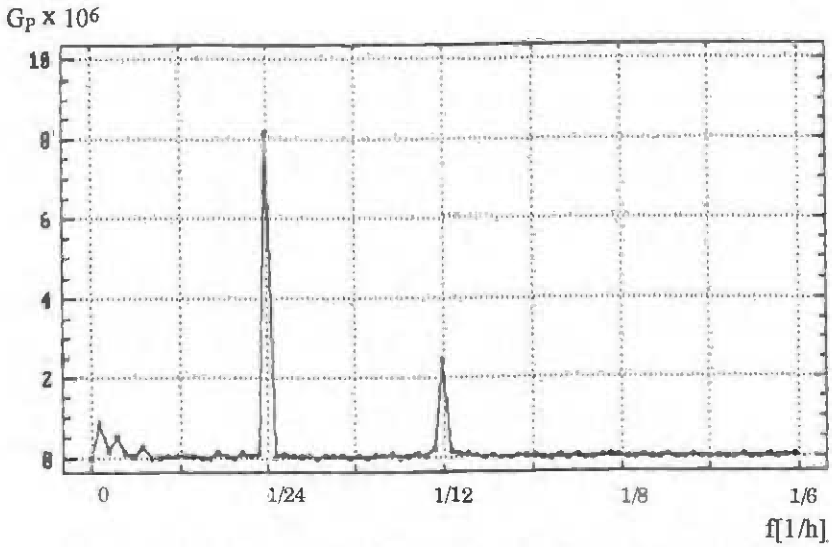


Fig. 4.8. The periodogram of the load process of the average voltage line

The weekly periodicity is exposed much lighter. It is the result of a great heterogeneity of the particular weeks caused by the different order of free and working Saturdays and other holidays.

The analysis of the load diagrams [78] assumes that three kinds of days in a week can be distinguished considering the 24-hour load profiles: holiday days, work days followed by the holiday day and other work days.

The series of the load values in the period of three weeks for the selected feeder of medium voltage at 18.00 o'clock for distinguished kinds of days are shown in Fig. 4.9.

This book generally considers the load models for work days. The models for the week load profile can be obtained, for example, by the joint of the models of different kinds of days in the order of their appearance [110].

The 24-hour variation of the load can be represented as the sum [37]

$$P(k) = P_p(k) + y(k), \qquad (4.74)$$

where:

$P_p(k)$ - the periodical component,

$y(k)$ - the random component.

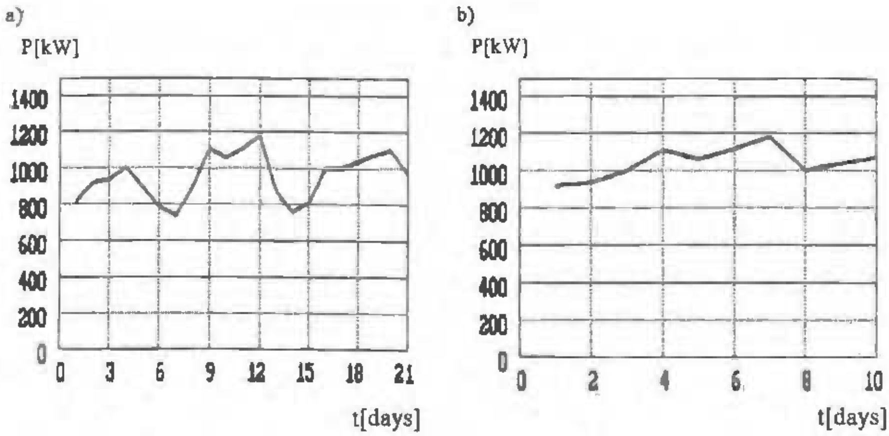


Fig. 4.9. The series of the load values of the medium voltage feeder at 18.00 o'clock for the distinguished kinds of day: a) all days of a week, b) work days

The periodical component of the 24-hour load variation is often presented in an analytical form as the finite Fourier series [37]

$$P_p(k) = \sum_{n=0}^{l} (a_n \sin\frac{2\pi n}{N} k + b_n \cos\frac{2\pi n}{N} k) \tag{4.75}$$

$n = 0, 1, \ldots, N-1,$

$k \leq N/2,$

where N is the number of the sample points during 24-hours.

The model parameters (4.75) are estimated on the basis of the criterion of the minimum of the weighted squares sum [37].

The spectral analysis of the 24-hour load processes at the buses of the medium voltage distribution system shows that regarding the periodical components of the frequency $f \leq 8/86400$ Hz in the model (4.75) is sufficient in most cases (in spite of the special receivers). The above results indicate that the reproduction of the periodical components of the load process in most of the practical cases needs the sampling with the $\Delta t = 90$ minutes interval.

The residue series y(k) presents the random load fluctuations caused by different random factors. This component is often modelled by the autoregressive model

$$y(k)=\sum_{i=1}^{p}\alpha_i y(k-i)+w(k)\,, \qquad (4.76)$$

where:

α_i - the model coefficients,

w(k) - the uncorrelated, zero-mean white noise.

The model coefficients are calculated on the basis of the autocorrelation function value $R_y(m)$ of the process (4.76) by solving the Yule-Walker set of equations [16].

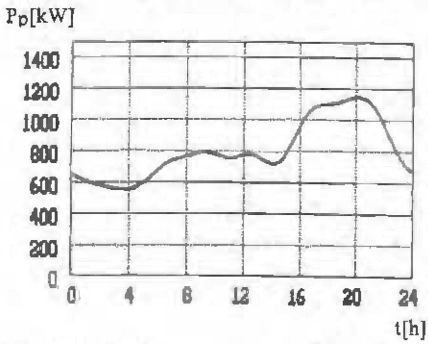


Fig. 4.10. Periodic component of load model

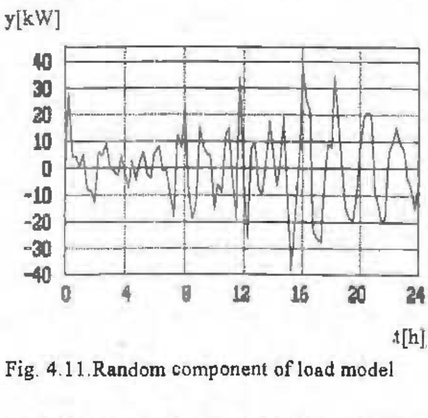


Fig. 4.11.Random component of load model

The model (5.76) adequacy evaluation can be done by the examination of residue series w(k) of the model. If the model is proper they have properties similar to the properties of white noise. The effective tool to detect the periodical deviations from randomness is the cumulated periodogram [16]. The significance of the residue periodogram deviations from its value for white noise may be evaluated by using the Kolmogorov-Smirnov test.

The analysis of the real load processes shows that good results are obtained by using the autoregressive models of the first order (p=1) or the second order (p=2).

Fig. 4.10÷4.13 present respectively: the periodic component $P_p(k)$ (for m=8) and the random component y(k) of the load model, the autocorrelation function for the residue series y(k), and the cumulated periodogram of the residue series w(k) (for p=1) for the 24-hour load process of the selected feeder of a medium voltage distribution system. Fig. 4.13 also shows the 5 and 25 percent confidence interval for the Kolmogorov-Smirnov test.

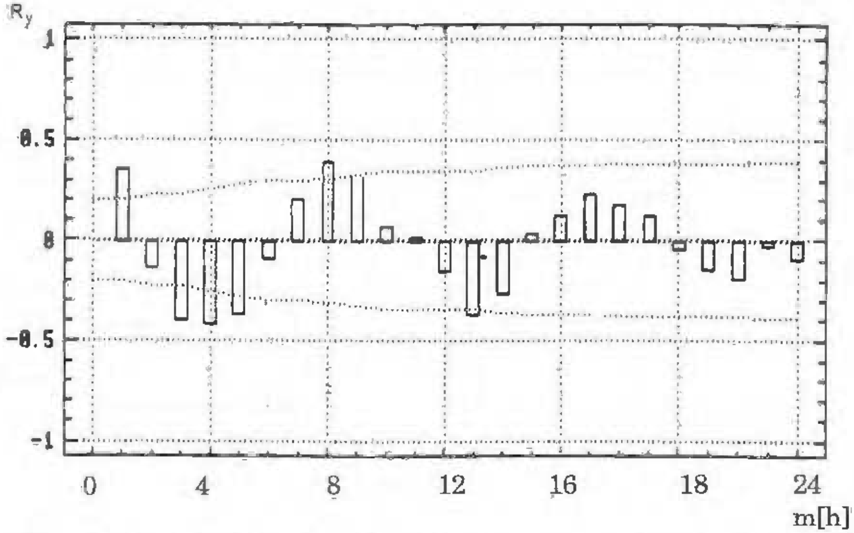


Fig. 4.12. Autocorrelation function of the random component y(k) of the load model

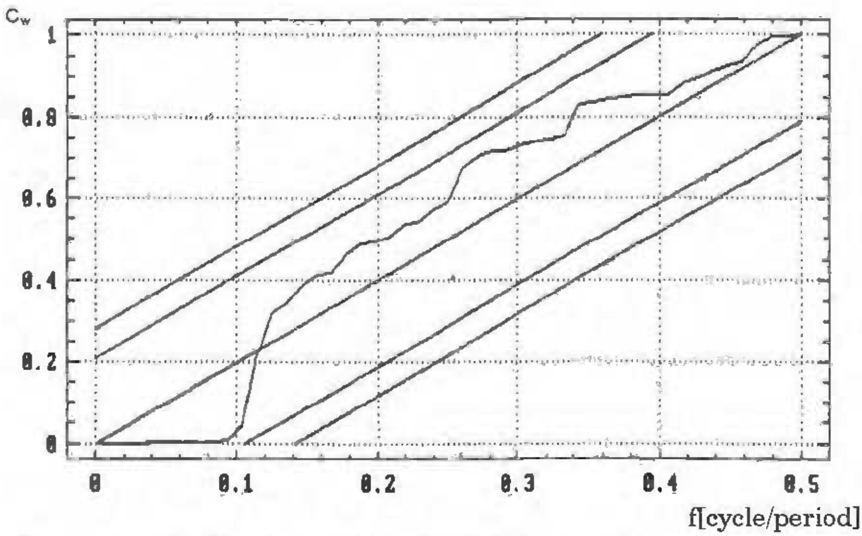


Fig. 4.13. Cumulated periodogram of residuals w(k) of the load model

4.3.2. The state vector dynamic estimation

The considerations made in section 4.2 concern the static estimation without taking into account the variation in the time of the vectors of observation, state, and observation errors. The model described by the equation (4.16) refers to the same time moment. The dynamic estimation takes into consideration the way of the forming of the state vector components in time. The purpose of dynamic estimation is to calculate the state estimate of the system at the moment k+1, using the measurement results obtained till the moment k. Under real time processing the result of these calculations should be known at the moment k.

The following initial model is suggested for the purpose of the dynamic estimation of the power distribution system state vector [96]

$$\begin{cases} \mathbf{x}(k+1) = \mathbf{F}(k)\mathbf{x}(k) + \mathbf{w}(k) \\ \mathbf{z}(k) = \mathbf{h}[\mathbf{x}(k),k] + \mathbf{v}(k), \\ k = 1,2,\dots \end{cases} \tag{4.77}$$

where:

- $\mathbf{x}(k)$ - n-dimensional state vector,
- $\mathbf{z}(k)$ - m-dimensional observation vector,
- $\mathbf{v}(k)$ - m-dimensional vector of the observation errors,
- $\mathbf{w}(k)$ - n-dimensional vector of the random disturbances at the input,
- k - the discrete time,
- $\mathbf{h}[\mathbf{x}(k),k]$ - m-dimensional vector of the prediction function,
- $\mathbf{F}(k)$ - m×n-dimensional transition matrix.

The statistic properties of the disturbance processes $\{w_k\}$ and $\{v_k\}$ are described by their moments of the first and the second order [96].

$$E[\mathbf{v}(k)] = 0 \tag{4.78}$$

$$E[\mathbf{v}(k_1)\mathbf{v}^T(k_2)] = \begin{cases} \mathbf{R}(k_1), & k_1 = k_2 \\ 0 & k_2 \neq k_2 \end{cases}, \tag{4.79}$$

$$E[\mathbf{w}(k)] = 0 \tag{4.80}$$

$$E[\mathbf{w}(k_1)\mathbf{w}^T(k_2)] = \begin{cases} \mathbf{Q}(k_1), & k_1 = k_2 \\ 0 & k_1 \neq k_2 \end{cases}, \tag{4.81}$$

The matrixes **R**(k) and **Q**(k) are non negatively defined and symmetric for all k.

The concept of the solution of the power distribution system state vector dynamic estimation problem is based on the linearization of the nonlinear model and then using the theory introduced for the linear models. The estimator obtained as the result of such an approach is called the extended Kalman filter [4].

The expansion of the function $\mathbf{h}[\mathbf{x}(k), k]$ in the Taylor series in the neighbourhood of the previous estimate $\hat{\mathbf{x}}(k/k-1)$ and the omission of the higher order terms of the series makes the approximation of the model (4.77) possible in the following form

$$\begin{cases} \mathbf{x}(k+1) = \mathbf{F}(k)\mathbf{x}(k) + \mathbf{w}(k) \\ \mathbf{z}(k) = \mathbf{H}(k)\mathbf{x}(k) + \mathbf{v}(k) + \mathbf{y}(k), \end{cases} \tag{4.82}$$

where $\mathbf{y}(k)$ is calculated from the equation

$$\mathbf{y}(k) = \mathbf{h}[\hat{\mathbf{x}}(k/k-1)] - \mathbf{H}(k)\hat{\mathbf{x}}(k/k-1). \tag{4.83}$$

In the formula:

$$\mathbf{H}(k) = \left.\frac{\partial \mathbf{h}[\mathbf{x}(k), k]}{\partial \mathbf{x}}\right|_{\mathbf{x}=\hat{\mathbf{x}}(k/k-1)} \tag{4.84}$$

where $\hat{\mathbf{x}}(k/k-1)$ denotes the estimate of the state vector at the moment k, calculated on the basis of the measurements made till the moment k-1.

As a result the one-step forecast is obtained.

For such approximation of the model (4.77) the extended Kalman filter is described by the following equations [4]

$$\hat{\mathbf{x}}(k+1/k) = \mathbf{F}(k)\hat{\mathbf{x}}(k/k), \tag{4.85}$$

$$\hat{\mathbf{x}}(k/k)\hat{\mathbf{x}}(k/k-1)\mathbf{L}(k)\{\mathbf{z}(k) - \mathbf{h}[\hat{\mathbf{x}}(k/k-1), \mathbf{k}]\}, \tag{4.86}$$

$$\mathbf{L}(k) = S(k/k-1)\mathbf{H}^T(k)W(k)^{-1}, \tag{4.87}$$

$$\Omega(k) = \mathbf{H}(k)\Sigma(k/k-1)\mathbf{H}^T + \mathbf{R}(k), \tag{4.88}$$

$$\Sigma(k+1/k) = \mathbf{F}(k)\Sigma(k/k)\mathbf{F}^T(k) + \mathbf{Q}(k), \tag{4.89}$$

$$\Sigma(k/k) = \Sigma(k/k-1) - \Sigma(k/k-1)\mathbf{H}^T(k) \times$$
$$\times[\mathbf{H}(k)\Sigma(k/k-1)\mathbf{H}^T(k) + \mathbf{R}(k)]^{-1}\mathbf{H}(k)\Sigma(k/k-1). \tag{4.90}$$

The initial conditions are expressed by

$$\Sigma(0/-1) = \mathbf{P}(-1), \tag{4.91}$$

$$\hat{\mathbf{x}}(0/-1) = \hat{\mathbf{x}}(-1). \tag{4.92}$$

The conditions P(- 1) and x (-1) are calculated from the following equations

$$\mathbf{P}(-1) = [\mathbf{H}^T(-1)\mathbf{R}^{-1}(-1)\mathbf{H}(-1)]^{-1}, \tag{4.93}$$

$$\hat{\mathbf{x}}(-1) = \bar{\mathbf{x}}(-1) + [\mathbf{H}^T(-1)\mathbf{R}^{-1}(-1)\mathbf{H}(-1)]^{-1} \times \times \mathbf{H}^T(-1)\mathbf{R}^{-1}(-1)\{\mathbf{z}(-1) - \mathbf{h}[\bar{\mathbf{x}}(-1)]\}, \tag{4.94}$$

where $\bar{\mathbf{x}}(-1)$ is the initial estimate of the state vector at the moment k = -1.

The dependencies (4.93) and (4.94) should be regarded as iterative, calculated till the moment when

$$|\mathbf{P}_l(-1) - \mathbf{P}_{l-1}(-1)| < \delta, \tag{4.95}$$

and

$$|\hat{\mathbf{x}}_l(-1) - \mathbf{x}_{l-1}(-1)| < \varepsilon, \tag{4.96}$$

where:

l - the iteration number,

ε, δ - a priori accepted precision of the estimate calculations for the state vector and for the covariance matrix respectively.

The form of the matrix F(k) is a result of the property of the periodicity of the power demand process. As was shown in the section 4.2.1, the system buses can be divided into classes considering the 24-hour load variation, and the typical 24-hour load profile can be attributed to the each class.

The typical load profiles are the basis for the forecast of the real loads in accordance with the dependence

$$\hat{P}_{i,j}(k+1/k) = \frac{\overline{P}_j(k+1)}{\overline{P}_j(k)}\hat{P}_{i,j}(k/k), \qquad (4.97)$$

where:

$\hat{P}_{i,j}(k+1/k), \hat{P}_{i,j}(k/k)$ – the load estimates at the moment k+1 and k respectively, calculated on the basis of the measurements done till the moment k, for the node i, which belongs to the class j,

$\overline{P}_j(k+1), \overline{P}_j(k)$ – the relative load value at the moment k+1 and k, respectively, according to the typical load profile for the customer class j.

The value of the voltage magnitude at the supplying bus may be forecasted on the basis of the operation program of the voltage regulator of the supplying transformer:

$$\hat{V}_0(k+1/k) = \frac{\overline{V}_0(k+1)}{\overline{V}_0(k)}\hat{V}_0(k/k), \qquad (4.98)$$

where:

$\hat{V}_0(k+1/k), \hat{V}_0(k/k)$ – the estimates of voltage magnitudes at the supplying bus at the moment k+1 and k respectively, calculated on the basis of the measurements done till the moment k,

$\overline{V}_0(k+1), \overline{V}_0(k)$ – the programmed values of the voltage magnitudes at the supplying bus at the moment k+1 and k respectively.

In relation to the above statement, the matrix **F**(k) has the form (4.99).

The matrix **R**(k) of the process covariance{**v**(k)} is stated on the basis of the knowledge of noise introduced by the symbolic indicator. The matrix **Q**(k) of the process covariance {**w**(k)} is stated on the basis of the estimation of variance of the typical load profiles (4.10).

$$
F(k)=\begin{bmatrix}
\frac{\overline{V}_0(k+1)}{\overline{V}_0(k)} & & & & & & 0 \\
& \frac{\overline{P}_{1,j_1}(k+1)}{\overline{P}_{1,j_1}(k)} & & & & & \\
& & \ddots & & & & \\
& & & \frac{\overline{P}_{i,j_i}(k+1)}{\overline{P}_{i,j_i}(k)} & & & \\
& & & & \ddots & & \\
0 & & & & & & \frac{\overline{P}_{n,j_n}(k+1)}{\overline{P}_{n,j_n}(k)}
\end{bmatrix} \tag{4.99}
$$

Having the above representation of matrices and vectors, the algorithm of the dynamic estimation of the state of the distribution system may be calculated on the computer. The estimation of the extended Kalman filter operation quality and accuracy in the conditions of the electrical power distribution system can be made by the computer simulation.

As in the section 4.2.2, to calculate the matrix $\mathbf{\Sigma H}^T[\mathbf{H\Sigma H}^T+\mathbf{R}]^{-1}$ we must use the indirect method which is more effective from the computational point of view.

1. Calculate

$$\mathbf{D} = \mathbf{\Sigma H}^T$$

2. Calculate

$$\mathbf{C} = \mathbf{H}(\mathbf{\Sigma H}^T)$$

3. Solve the linear equation

$$\mathbf{XC} = \mathbf{D}$$

using the method of the factorization of the square root.

To fulfil the limitations (4.60) we have to use the procedure described by the dependencies (4.62)÷(4.64).

5. THE APPLICATION OF THE STATE ESTIMATION METHODS TO THE POWER DISTRIBUTION SYSTEM CALCULATIONS

5.1. The static state estimation

The methods of the system state estimation worked out in the chapter 4 need the final verification before further utilization. To verify the method of the static estimation of the state vector, computer simulation was used. For that purpose the simulation experiment described in the section 3.4 was extended. The calculation of the power flow in the system is preceded each time by determination of the system state vector estimate. The calculated state vector estimate is used to determine the estimates of all quantities defined in the power flow study. Other assumptions and conditions of the experiment realization were not changed. Simulation calculations were made under the following algorithm.

1. Generate for the selected bus i a random number δ_{P_i} of beta distribution BT(-90, 900, 1,275, 12,75).
2. Calculate P_i and Q_i according to the equations (3.48) and (3.49).
3. Check whether the value P_i is within the range

   $$0.1S_{ni} \le P_i \le S_{ni}$$

   If so, go to the step 4, and if not, come back to the step 1.
4. Check whether all buses (i=n) were taken into account. If not, choose the next bus i+1 and go to the step 1, and if so, go to the step 1.
5. Calculate the estimate of the power distribution system state vector.
6. Calculate, on the basis of the state vector estimate, the power flow in the system.
7. Calculate the values of other functions described for the system branches and buses.
8. Write down the results of the calculations.
9. Check whether the required number of simulations was done. If not, choose the bus i=1 and go to the step 1, and if so, finish the calculations.

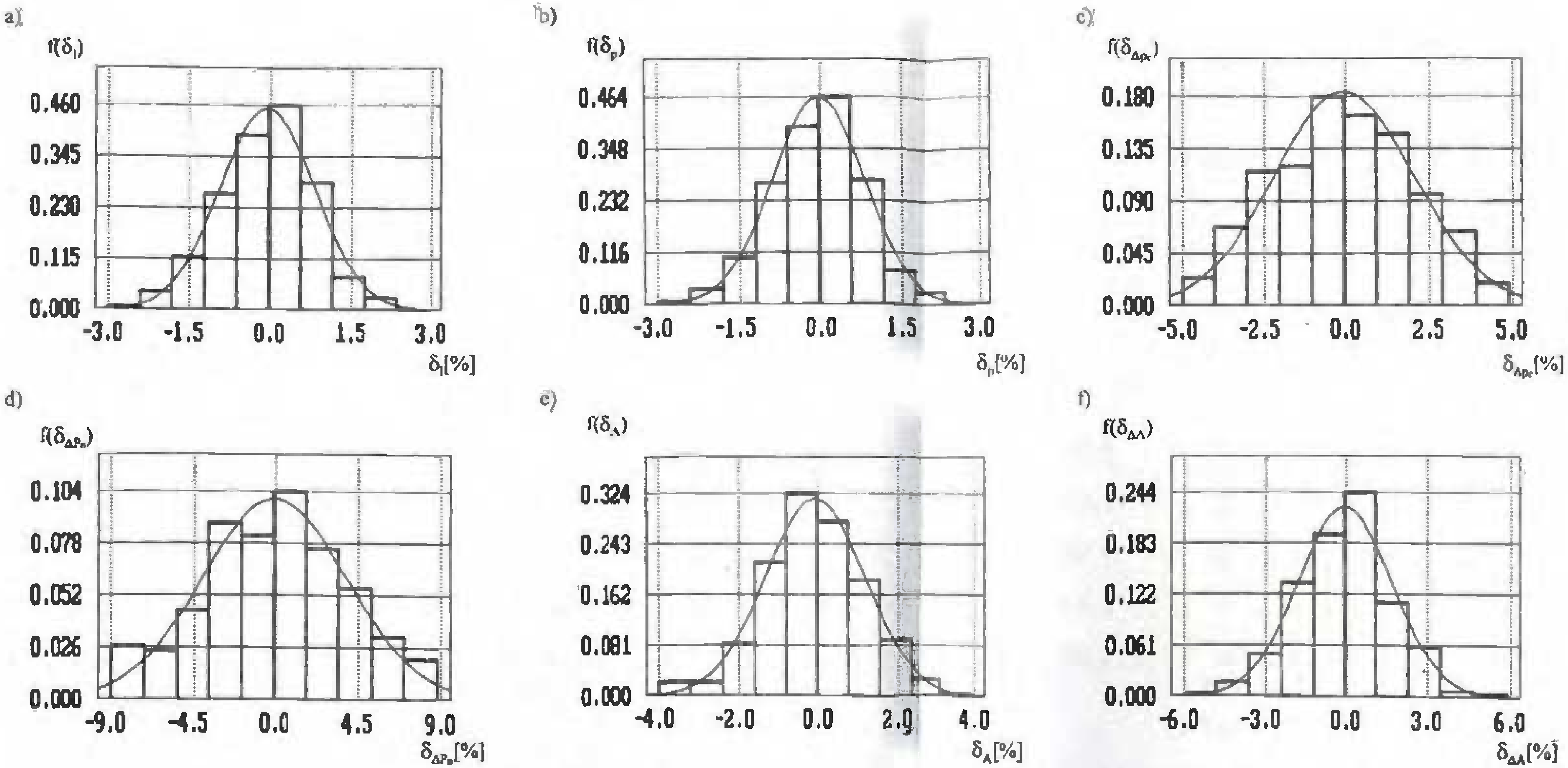


Fig. 5.1. Histograms and probability density functions for percentage errors of calculations: a) current amplitude in supplying transformer, b) active power supplying bus, c) total active power losses, d) total load losses, e) daily energy consumption, f) daily active energy losses

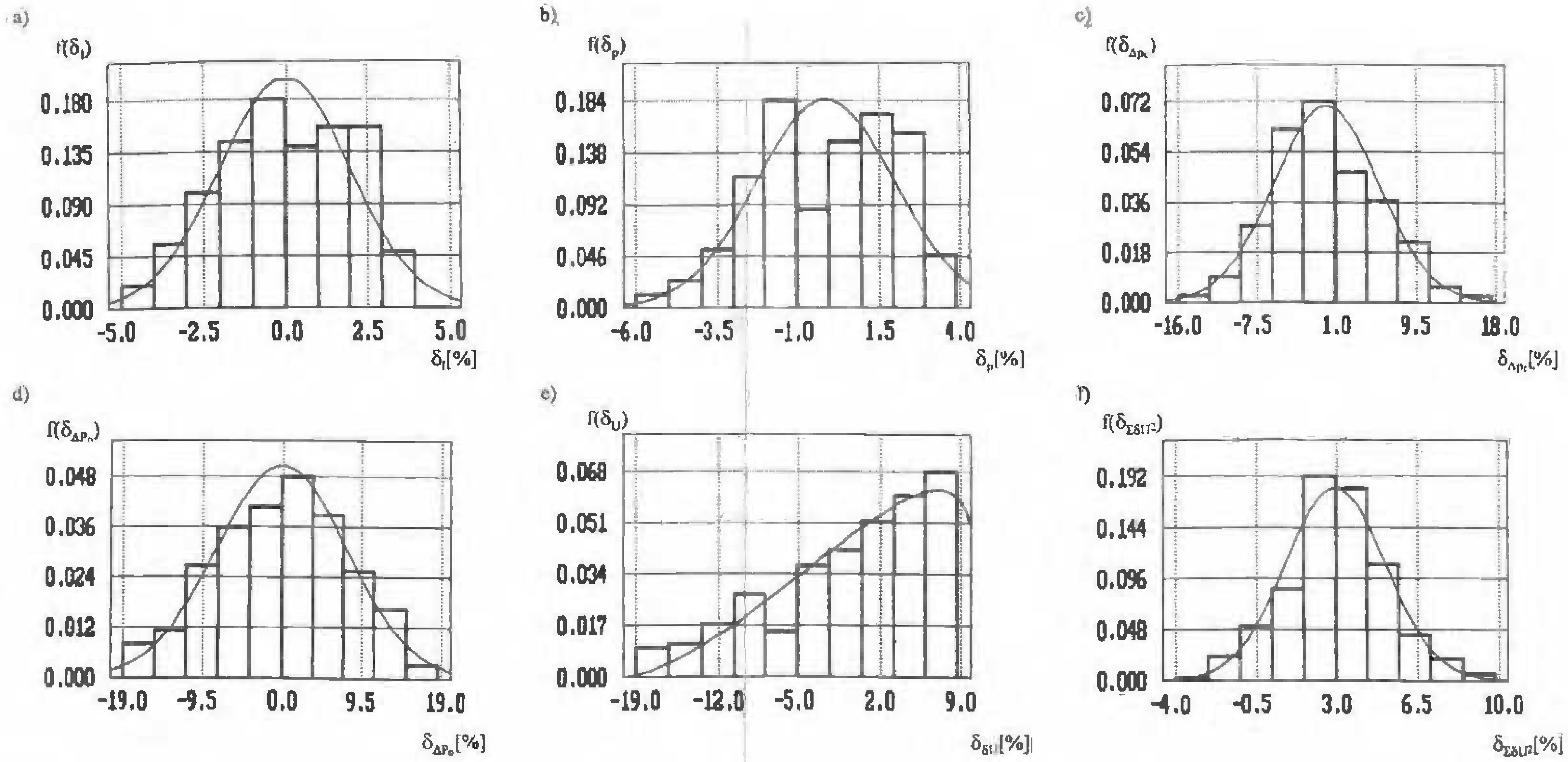


Fig. 5.2. Histograms and probability density functions for percentage errors of calculations: a) current amplitude flows in line L1, b) active power flows in line L1, c) total active power losses in the part of the system supplied from line L1, d) load active power losses in the part of the system supplied from line L1, e) voltage deviation at bus PS428, f) sum of voltage square deviation on LV receiving buses

Table 5.1.

Estimation of parameters and results of test of goodness of normal distribution fit with errors distribution

Output quantity	Distribution parameters and critical value of significance level	Investigated object								
		Transformer TR1	Line L1	Line L11	Line L29	Line L39	Line L40	Line L60	Line L77	Line L93
I	m	0.03	0.01	0.01	-0.80	0.05	0.02	0.02	0.07	0.08
	δ	0.88	2.01	1.04	3.21	0.52	0.92	3.77	3.72	0.54
	α	0.99	0.31	0.07	0.05	0.00	0.22	0.35	0.30	0.00
P	m	0.02	0.09	0.02	-0.09	0.09	0.15	0.01	-0.02	0.02
	δ	0.86	2.15	0.80	3.27	0.50	0.95	3.64	3.71	0.52
	α	0.99	0.06	0.09	0.08	0.00	0.25	0.34	0.31	0.00
Q	m	0.12	0.03	0.01	-0.03	0.04	0.07	0.10	0.17	0.06
	δ	3.41	3.52	2.49	5.83	2.62	2.24	5.77	5.23	0.88
	α	0.41	0.19	0.09	1.00	0.00	0.99	0.06	0.99	0.00
ΔP	m	0.02	0.07	0.04	0.04	-0.01	0.03	0.03	-0.05	0.04
	δ	2.16	5.66	3.31	4.68	4.59	3.16	5.60	5.24	4.13
	α	0.99	0.99	0.16	0.99	0.00	0.99	1.00	1.00	0.00
ΔPL	m	0.05	0.12	0.94	0.44	0.00	0.02	0.02	-0.05	0.08
	δ	5.11	13.19	10.37	14.65	0.00	11.07	8.76	16.52	1.07
	α	0.99	0.99	1.00	0.06	-	0.99	0.99	0.13	0.00
ΔPT	m	0.15	0.08	0.06	-0.22	-0.01	0.04	0.04	-0.05	0.02
	δ	1.61	3.67	3.36	4.24	4.59	3.12	5.46	4.65	5.41
	α	0.99	1.00	0.34	0.99	0.00	0.38	0.36	1.00	0.00
A	m	0.03	0.03	0.08	0.02	0.05	0.02	0.08	0.01	0.02
	δ	1.26	3.20	2.55	3.17	0.48	3.11	4.39	5.40	0.05
	α	1.00	0.51	0.06	0.17	0.00	0.99	0.17	1.00	0.00
ΔA	m	0.01	0.06	0.09	-0.03	-0.46	0.03	0.02	-0.02	0.02
	δ	1.76	3.67	3.76	3.87	3.43	4.74	4.96	5.29	2.76
	α	0.99	0.99	0.06	0.49	0.00	0.11	1.00	0.99	0.00

Table 5.2.

Quotients of standard deviations of relative errors of calculations of investigated quantities without and with state estimation

Output quantity	Investigated object								
	Transformer TR1	Line L1	Line L11	Line L29	Line L39	Line L40	Line L60	Line L77	Line L93
	σ_1/σ_2								
I	8,4	10,6	12,9	8,4	77,6	14,7	3,1	5,2	67,1
P	8,5	9,8	16,5	8,3	77,9	14,1	3,2	5,2	68,4
Q	5,6	6,6	6,4	3,8	28,2	6,8	2,9	3,9	44,6
ΔP	4,0	3,7	5,8	5,8	12,0	4,8	3,1	3,6	11,9
ΔPL	3,0	3,5	3,3	3,9	oo	3,4	3,3	3,6	65,9
ΔPT	4,0	5,2	4,1	5,9	9,4	3,8	2,7	3,3	7,6
A	6,7	6,1	5,4	8,1	81,0	4,6	3,7	3,5	711,4
ΔA	4,4	3,1	4,3	3,7	15,4	3,0	3,1	3,0	12,2

Calculations

The fifth step of the algorithm should be explained in more detail. It is assumed that the remote measurements of the following values are accessible:
- the voltage magnitude at the supplying bus,
- the magnitudes of the current flow in the particular feeders,
- the active power flow in the supplying transformer,
- the passive power flow in the supplying transformer.

The value of the active power received at the system buses is calculated each time according to the dependence (3.48).

The elements r_i of the weight matrix R^{-1} are assumed on the basis of the estimation of the particular observation error variances. The following values were assigned to the corresponding measurements:

The measurement of the voltage magnitude at the supplying bus

$$r_{V_0} = (\frac{1}{3} k_{V_0} \frac{V_{max}}{V_0})^2 \qquad (5.1)$$

The measurement of the magnitudes of the currents in the particular feeders

$$r_{I_l} = (\frac{1}{3} k_{I_l} \frac{I_{l\,max}}{I_l})^2 \qquad (5.2)$$

The measurement of the active power flow in the supplying transformer

$$r_{p_T} = (\frac{1}{3} k_{p_T} \frac{p_{t\,max}}{p_T})^2 \qquad (5.3)$$

The measurement of the passive power flow in the supplying transformer

$$r_{q_T} = (\frac{1}{3} k_{q_T} \frac{q_{t\,max}}{q_T})^2 \qquad (5.4)$$

The estimation of the active power received at the system buses

$$r_{P_w} = (0.70 \frac{P_w}{P_{n,w}})^2 \qquad (5.5)$$

In the equations the accuracy class of the set of measurement devices for measurement of the particular quantity is marked by the letter "k" with the proper index. The boundary values of the measuring ranges of the particular measuring instruments are marked by the index "max".

The values of the β coefficient (4.61) representing the width of the limiting zone and the weight increasing coefficient λ (4.64) were assumed as equal $\beta=0,05$ and $\lambda=10$ properly. The iterative calculations were repeated, according to the dependence (4.37), till the moment when the assumed precision $\varepsilon=10^{-2}$ was obtained. Other calculations were performed according to the description in the section 3.4.

As a result of the simulation experiment the form and the parameter values of the probability distributions of the relative errors of system calculation, with the utilization of the system state vector estimation, were estimated.

The results of the simulation investigations are presented graphically as histograms of the observed error values (Fig. 5.1 and Fig. 5.2). The same figures show the diagrams of the function of the theoretical distribution density selected for the experimental data representation.

Table 5.1 presents the estimation of the parameters and the results of the test of goodness of the normal distribution fit for the distributions of relative errors made during the calculation of the investigated quantities using the static estimation of the state vector. The results of the tests of goodness show that these distributions may be approximated by the normal distributions with a great accordance. Only in the case of two feeders (L39 and L93), which supply only two substations each, there is not a reason to accept a hypothesis that those distributions are normal. The hypothesis that all average values come from the samples which belong to the populations of the same expected value and may be regarded as zero was verified, on the significance level $\alpha = 0,05$, using the average homogeneity test and the significance of expected value test [108, 115]. The results of the variance homogeneity test [67] show that there are no reasons, on the significance level $\alpha = 0,001$, to accept the hypothesis that the estimations of the variance come from the samples of the populations of the same variance.

Table 5.2 shows the quotients of standard deviations of relative errors of calculations of the investigated quantities without and with consideration of the state vector static estimation.

Table 5.3 presents the same data for the voltage deviations at the selected system buses and the sum of the square of the voltage deviations at the distribution buses.

Table 5.3

Estimation of parameters and results of tests of goodness of normal distribution fit with error distributions for voltage deviation calculated using state estimation

Investigated object	Mean value	Standard deviation	Critical value of significance level	Quotients of standard deviation without and with state estimation
	m	δ	α	δ_1/δ_2
PS 149	1.3	4.2	0.02	2.0
PS 25A	0.7	9.9	0.17	2.2
PS 268	0.9	8.5	0.06	1.4
PS 288	-12.7	13.7	0.01	2.0
PS 428	-0.4	7.2	0.01	1.4
PS 437	-0.1	10.6	0.99	2.2
PS 64	-6.8	5.1	0	1.5
$\Sigma\delta U^2$	3.0	2.2	0.99	2.0

In more cases (6 MV/LV substations) the hypothesis that the distributions of the errors of the voltage deviation at the system buses are well approximated by the normal distributions can be accepted on the significance level $\alpha = 0{,}01$. The homogeneity tests of the average values and of variances gave unsatisfactory results.

The distribution of the relative errors of the square sum of voltage deviations at distribution buses is very well approximated by the normal distribution.

The comparison of the standard deviations of relative errors made during the calculation of the investigated output quantities without consideration and with consideration of the system state vector estimation, shows that the suggested method of the state vector static estimation is very effective. The considerable reduction (from several to tens times) of standard deviations of calculation errors were obtained.

The results of the simulation experiment show that the application of the proposed method of the static state vector estimation to the power distribution system calculations makes determination of system states possible, with practically sufficient accuracy even in the case of great uncertainty of data on loads at system buses.

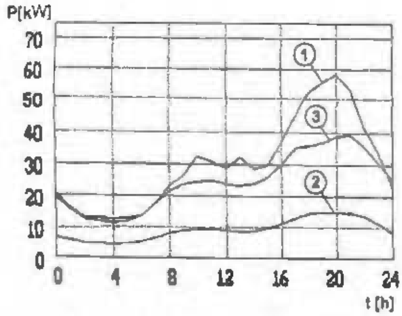
5.2. The dynamic state estimation

To verify practicability of the state vector dynamic estimation method proposed in the section 4.3 to the power distribution system calculations the special computational and measuring experiment was designed and implemented. The simultaneous registration of the 24-hour passages of: active and passive power flow in the 110/15 kV supplying transformer, the voltage magnitude at the 15 kV busbar system, the current magnitudes in 15 kV feeders, and the active load received at the twenty selected MV/LV substations were made in the part of the existing electric power distribution system of medium voltage (see Appendix). This data was used for calculations and verifications of the method of the system state vector dynamic estimation.

Calculations

The calculation was made for the one-hour forecast. It was assumed that the remote measurements of the following values are accessible at each hour:

- the voltage magnitude at the supplying bus,
- the magnitudes of the currents in the MV feeders,
- the active power flow in the supplying transformer,
- the passive power flow in the supplying transformer.



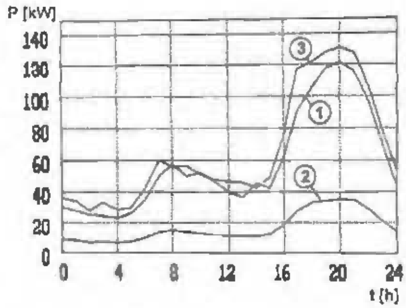


Fig. 5.3. Results of dynamic estimation of 24-hour load profile at the substation PS1195

Fig. 5.4. Results of dynamic estimation of 24-hour load profile at the substation PS377

1 - measured load profile, 2 - preliminary estimated load profile, 3 - on-line estimated load profile

The components P_W (the active power received at the system buses) of the observation vector **z** (4.18) are calculated for each hour k on the basis of the typical load profiles assigned to each bus, and on the basis of the initial dispatchers estimation of peak load at these buses.

Using such defined observation vector, 24 state vector elements were calculated for the successive hours of a day according to the equations (4.85)÷(4.90). In such a way the estimates of 24-hour load profile forecasted with one-hour lead were obtained for the distribution buses. Fig. 5.3 and Fig. 5.4 show the examples of the results. Each figure shows three diagrams: the measured 24-hour load profile, the profile which is the result of the typical 24-hour load profile and the dispatcher estimation of the peak-load, and the profile obtained as the result of the state vector dynamic estimation.

To verify the proposed method of the system state vector dynamic estimation, the values of the relative mean-square deviations (4.10) between the real load profiles and the profiles resulted from the typical 24-hour load profiles and the dispatcher peak-load estimation, and the profiles received as a result of the dynamic estimation calculations were compared for 20 selected substations. The results of the calculations are presented in the Table 5.4.

The results of the experiments show the great efficiency of the worked out method of the distribution system state vector dynamic estimation. In the investigated sample, the double reduction of the sum of relative mean-square deviations in relation to the originally assumed profiles was obtained. The values of mean-square deviations were reduced at most of the controlled system buses. The value of deviation was a slightly larger only at two distribution buses (PS-234 and PS-170).

Table 5.4

The values of the relative mean-square deviations between the real 24-hour load profiles and the profiles resulted from the typical 24-hour load profiles and the dispatcher estimation of the peak load (η_1), and the profiles received as a result of the dynamic estimation calculations (η_2).

Number	Substation name	Relative mean-square deviation	
		Before estimation η_1	After estimation η_2
1	PS 437	0.66	0.04
2	PS 234	0.33	0.36
3	PS 170	0.30	0.38
4	PS 1243	0.56	0.39
5	PS 651	0.40	0.28
6	PS 485	0.89	0.50
7	PS 1195	0.78	0.20
8	PS 1326	0.88	0.60
9	PS 268	0.47	0.41
10	PS 550	0.75	0.30
11	PS 67	0.59	0.33
12	PS 629	0.70	0.13
13	PS 25	0.44	0.30
14	PS 1237	0.92	0.25
15	PS 746	0.89	0.14
16	PS 261	0.79	0.43
17	PS 288	0.69	0.14
18	PS 471	0.50	0.34
19	PS 377	0.81	0.10
20	PS 315	0.78	0.26
Sum of deviations		13.14	6.18

It should be said that those results were obtained with the assumption of the reduced set of remote measurements. Each supplement of the observation vector by additional remote measurement would make the results better.

The knowledge of the system operation state is the basis for determination of the system operation optimal control. The operation optimization of the power and energy losses and the voltage levels are made at the distribution systems, generally by the proper system configuration and by the voltage regulation with the HV/MV and MV/LV transformers [22, 27, 46, 62, 104].

The best known algorithms of the optimization of the system configuration and the voltage regulation described in the book [55] are based on the assumption of the knowledge of the loads at the system buses. As it was justified in the chapter 3, implementation of those optimization methods have had till now no practical reason because of the great uncertainty of the input data on load. As a rule, theoretical effects of optimization are contained in confidence area of calculations. The results confirm the thesis that calculations used to optimize distribution system performance depend upon the load estimation. The implementation of the estimation methods worked out in this book makes it possible to determine the system state vector with an acceptable accuracy. The state vector is then the basis for distribution system calculations.

6. COMPUTER PROGRAMS FOR THE POWER DISTRIBUTION SYSTEM STATE ESTIMATION

In the framework of the studies on the methods of the electric power distribution system state estimation, dozens of computer programs for the numerical analysis of the experimental data and for electric power distribution system calculations were prepared. The calculations were done on the IBM PC/386 computer. Most of the programs were written in Turbo Pascal programming language [66].

Description of the programs

1. DIGIT.EXE. The program is used to convert the graphical data (the diagrams produced on a paper tape by the self-writing recorders) to real numbers which correspond to the measurement values. This conversion is made with the use of the KD 4080 digitizer. The converted data is written to the data base on a hard disk.
2. VIEW.EXE. The program enables graphical presentation of the results of the measurements on a monitor screen or on a printer.
3. STAT.EXE. The program enables many statistic calculations and verification of the statistic hypothesis. The results can be presented in text and graphic format.
4. TSA.EXE. The program enables calculations connected with the numerical analysis of time series. The results can be presented in text and graphic format.
5. KLASY.EXE. The program is a practical implementation of the algorithm of the customer division into classes according to the 24-hour load profiles (see section 4.2).
6. TGO.EXE. The program is a practical implementation of the algorithm of the construction of the typical 24-hour load profile for the particular classes of customers (see section 4.2).
7. ELCAD.EXE. The program uses the OrCAD program package for which the libraries of the power distribution system elements were designed. The program enables graphical mapping of the system elements and configuration. The data is transformed from the graphic format to the topologically arranged graph, which is the basis for the operation of all system calculation algorithms

8. OBL-SIEC.EXE. The program enables the following system calculations: the calculation of the current and power flows, the voltage calculations, the calculations of the power and energy losses.
9. EXP-SIEC.EXE. The program is a practical implementation of the algorithm of the examination of the significance of the investigated quantities on the results of the electrical power distribution system calculations (see section 3.3).
10. SYM-SIEC.EXE. The program enables the simulation calculations to examine the influence of uncertainty of the evaluation of the peak-loads at the distribution buses on the results of the electrical power distribution system analysis. It is a practical implementation of the algorithm described in the section 3.4.
11. EST-SIEC.EXE. The program, written in Turbo C programming language, is a practical implementation of the algorithm of the static estimation of the power distribution system state vector (see section 4.2.2).
12. DYN-SIEC.EXE. The program, written in Turbo C programming language, is a practical implementation of the algorithm of the dynamic estimation of the electrical power distribution system state vector (see section 4.3.2).

The computer programs, worked out on the basis of the algorithms presented in the book, allow the static and dynamic estimation of the steady operation states of the electrical power distribution systems in real time. Due to the speed of calculations (about tenths of seconds), it is possible to utilize the calculation results to control the system operation in the on-line mode.

7. CONCLUSIONS

On the basis of the analysis of the results obtained in the book the following general conclusions are formulated:

1. The application of the theory of estimation enables real time estimation of the electrical power distribution system operation states with, practically sufficient accuracy. The estimation is made on the basis of available incomplete primary information on loads and customers, with use of the statistic compensation of the deficiency of remote measurements.
2. The most important point in power distribution system calculations is proper evaluation of active loads at the system buses.
3. Great errors in load evaluation existing in operational practice have a fundamental effect on the accuracy of computations made on a system and as a result there is not any explanation nowadays of using most of the known methods of the optimization of distribution system operation states.
4. The results of the measurements and of the analysis of the 24-hour load profiles of MV/LV substation in the distribution system, indicate the possibility of grouping distribution buses into characteristic classes of the similar load profiles, according to the structure of customers supplied from those buses.
5. The model of the 24-hour load passage at the system buses can be presented in a form of the sum of the periodical component described by the finite Fourier series, and the random component described by the autoregressive model.
6. Nowadays, the proper solving of the optimization problems in electrical power distribution systems should be based on the probabilistic and statistic methods and on the theory of stochastic processes.
7. The result of the simulation and measuring experiments confirmed great efficiency of the proposed methods of static and dynamic estimation of the power distribution system state vector.
8. The implementation of the worked out algorithms and programs in power utility dispatcher centers to the computer aided control of distribution systems operation allow better identification of the system operation states, improve the system control efficiency, and improve the quality of energy supplied to customers.

In the author's opinion the original achievements of this book are:

- the worked out information structure of the real time control system of the electrical power distribution system operation,
- the application and the adaptation of the theory of experimental design to the power distribution system conditions to build and experimentally verify the qualitative model of distribution system,
- the worked out methods of the division of customers into characteristic classes and of construction of the typical load profiles for such classes,
- the description of the stochastic properties of the load variation at the distribution system buses,
- the application and adaptation of the theory of estimation to the electric power distribution system conditions,
- the application and adaptation of the Kalman filter theory to the electric power distribution system conditions,
- the numerical solving of the derived equations of estimators of the system state vector in a static and dynamic cases,
- the demonstration of the efficiency of implementation of probabilistic and statistic methods in the analysis and control of the electric power distribution system operation.

The theoretical basis for the system of the estimation of the power distribution system operation states were worked out. This system may be the basis for the real time control of distribution system operation. The worked out computers programs can be implemented in the power utility dispatcher centers.

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